

# Lecture Notes on

# HEAT TRANSFER (18ME63)

## Department of Mechanical Engineering

### Program Outcomes

1. **Engineering knowledge:** Apply the knowledge of mathematics, science, engineering fundamentals, and an engineering specialization to the solution of complex engineering problems.
2. **Problem analysis:** Identify, formulate, review research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences.
3. **Design/development of solutions:** Design solutions for complex engineering problems and design system components or processes that meet the specified needs with appropriate consideration for the public health and safety, and the cultural, societal, and environmental considerations.
4. **Conduct investigations of complex problems:** Use research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions.
5. **Modern tool usage:** Create, select, and apply appropriate techniques, resources, and modern engineering and IT tools including prediction and modeling to complex engineering activities with an understanding of the limitations.
6. **The engineer and society:** Apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal and cultural issues and the consequent responsibilities relevant to the professional engineering practice.
7. **Environment and sustainability:** Understand the impact of the professional engineering solutions in societal and environmental contexts, and demonstrate the knowledge of, and need for sustainable development.
8. **Ethics:** Apply ethical principles and commit to professional ethics and responsibilities and norms of the engineering practice.
9. **Individual and team work:** Function effectively as an individual, and as a member or leader in diverse teams, and in multidisciplinary settings.
10. **Communication:** Communicate effectively on complex engineering activities with the engineering community and with society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions.
11. **Project management and finance:** Demonstrate knowledge and understanding of the engineering and management principles and apply these to one's own work, as a member and leader in a team, to manage projects and in multidisciplinary environments.
12. **Life-long learning:** Recognize the need for, and have the preparation and ability to engage in independent and life-long learning in the broadest context of technological change.

## Department of Mechanical Engineering

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**Subject: Heat Transfer**

**Subject Code: 18ME63**

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## Module 1: INTRODUCTORY CONCEPTS AND DEFINITIONS

**Introduction:** - We recall from our knowledge of thermodynamics that heat is a form of energy transfer that takes place from a region of higher temperature to a region of lower temperature solely due to the temperature difference between the two regions. With the knowledge of thermodynamics, we can determine the amount of heat transfer for any system undergoing any process from one equilibrium state to another. Thus, the thermodynamics knowledge will tell us only how much heat must be transferred to achieve a specified change of state of the system. But in practice we are more interested in knowing the rate of heat transfer (i.e. heat transfer per unit time) rather than the amount. This knowledge of rate of heat transfer is necessary for a design engineer to design all types of heat transfer equipment like boilers, condensers, furnaces, cooling towers, dryers etc. The subject of heat transfer deals with the determination of the rate of heat transfer to or from a heat exchange equipment and also the temperature at any location in the device at any instant of time.

The basic requirement for heat transfer is the presence of a “temperature difference”. The temperature difference is the driving force for heat transfer, just as the voltage difference for electric current flow and pressure difference for fluid flow. One of the parameters, on which the rate of heat transfer in a certain direction depends, is the magnitude of the temperature gradient in that direction. The larger the gradient higher will be the rate of heat transfer.

**Heat Transfer Mechanisms:** - There are three mechanisms by which heat transfer can take place. All the three modes require the existence of temperature difference. The three mechanisms are: (i) conduction, (ii) convection and (iii) radiation

**Conduction:** - It is the energy transfer that takes place at molecular levels. Conduction is the transfer of energy from the more energetic molecules of a substance to the adjacent less energetic molecules as a result of interaction between the molecules. In the case of liquids and gases conduction is due to collisions and diffusion of the molecules during their random motion. In solids, it is due to the vibrations of the molecules in a lattice and motion of free electrons.

**Fourier's Law of Heat Conduction:** - The empirical law of conduction based on experimental results is named after the French Physicist Joseph Fourier. The law states that the rate of heat flow by conduction in any medium in any direction is proportional to the area normal to the direction of heat flow and also proportional to the temperature gradient in that direction. For example, the rate of heat transfer in x-direction can be written according to Fourier's law as

$$Q_x \propto - A (dT / dx) \dots\dots\dots (1.1)$$

Or 
$$Q_x = - k A (dT / dx) W \dots\dots\dots (1.2)$$

In equation (1.2),  $Q_x$  is the rate of heat transfer in positive x-direction through area A of the medium normal to x-direction,  $(dT/dx)$  is the temperature gradient and k is the constant of proportionality

and is a material property called "*thermal conductivity*". Since heat transfer has to take place in the direction of decreasing temperature,  $(dT/dx)$  has to be negative in the direction of heat transfer. Therefore, negative sign has to be introduced in equation (1.2) to make  $Q_x$  positive in the direction of decreasing temperature, thereby satisfying the second law of thermodynamics. If equation (1.2) is divided throughout by  $A$  we have

$$q_x = (Q_x / A) = -k (dT / dx) \quad \text{W/m}^2 \dots \dots \dots (1.3)$$

$q_x$  is called the *heat flux*.

**Thermal Conductivity:** - The constant of proportionality in the equation of Fourier's law of conduction is a material property called the thermal conductivity. The units of thermal conductivity can be obtained from equation (1.2) as follows:

Solving for  $k$  from Eq. (1.2) we have  $k = -q_x / (dT/dx)$

Therefore, units of  $k = (W/m^2) (m/ K) = W / (m - K)$  or  $W / (m - ^\circ C)$ . Thermal conductivity is a measure of a material's ability to conduct heat. The thermal conductivities of materials vary over a wide range.

It can be seen from this figure that the thermal conductivities of gases such as air vary by a factor of  $10^4$  from those of pure metals such as copper. The kinetic theory of gases predicts and experiments confirm that the thermal conductivity of gases is proportional to the *square root of the absolute temperature*, and inversely proportional to the *square root of the molar mass  $M$* .

In the case of solids heat conduction is due to two effects: the vibration of lattice induced by the vibration of molecules positioned at relatively fixed positions, and energy transported due to the *motion of free electrons*. The relatively high thermal conductivities of pure metals are primarily due to the electronic component. The lattice component of thermal conductivity strongly depends on the way the molecules are arranged. For example, diamond, which is highly ordered crystalline solid, has the highest thermal conductivity at room temperature.

Unlike metals, which are good electrical and heat conductors, *crystalline solids* such as diamond and semiconductors such as silicon are good heat conductors but poor electrical conductors. Hence such materials find widespread use in electronic industry. Despite their high price, diamond heat sinks are used in the cooling of sensitive electronic components because of their excellent thermal conductivity. Silicon oils and gaskets are commonly used in the packaging of electronic components because they provide both good thermal contact and good electrical insulation.

**Convection:** - Convection heat transfer is composed of *two mechanisms*. Apart from energy transfer due to *random molecular motion*, energy is also transferred due to *macroscopic motion* of the fluid. Such motion in presence of the temperature gradient contributes to heat transfer. Thus, in convection the

total heat transfer is due to random motion of the fluid molecules together with the bulk motion of the fluid, the major contribution coming from the latter mechanism. Therefore, bulk motion of the fluid is a necessary condition for convection heat transfer to take place in addition to the temperature gradient in the fluid. Depending on the force responsible for the bulk motion of the fluid, convective heat transfer is classified into “forced convection” and “natural or free convection”. In the case of forced convection, the fluid flow is caused by an external agency like a pump or a blower where as in the case of natural or free convection the force responsible for the fluid flow (normally referred to as the buoyancy force) is generated within the fluid itself due to density differences which are caused due to temperature gradient within the flow field. Regardless of the particular nature of convection, the rate equation for convective heat transfer is given by

$$q = h \Delta T \dots\dots\dots(1.5)$$

where  $q$  is the heat flux,  $\Delta T$  is the temperature difference between the bulk fluid and the surface which is in contact with the fluid, and ‘ $h$ ’ is called the “convective heat transfer coefficient” or “surface film coefficient”. Eq.(1.5) is generally referred to as the Newton’s law of cooling.If  $T_s$  is the surface temperature ,  $T_f$  is the temperature of the bulk fluid and if  $T_s > T_f$ , then Eq. (1.5) in the direction of heat transfer can be written as

$$q = h [T_s - T_f] \dots\dots\dots(1.6a)$$

and if  $T_s < T_f$ , the equation reduces to

$$q = h [T_f - T_s] \dots\dots\dots(1.6b)$$

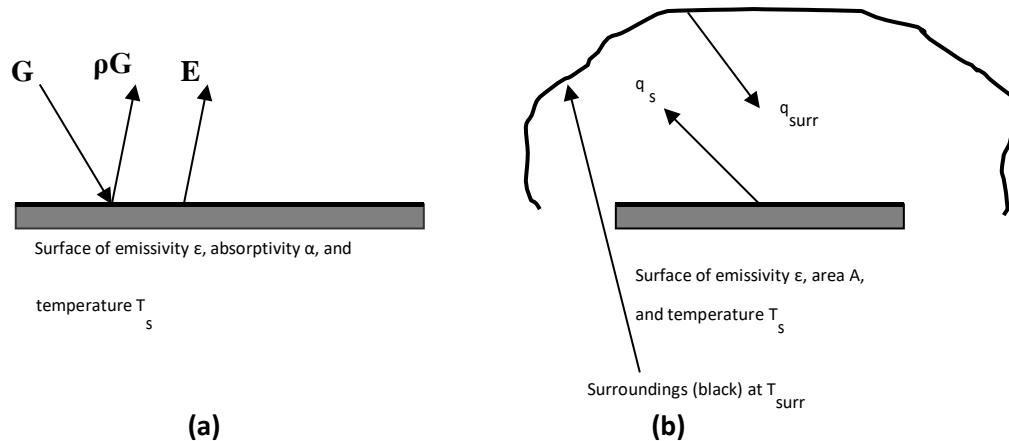
The heat transfer coefficient  $h$  depends on (i) the type of flow (i.e. whether the flow is laminar or turbulent), (ii) the geometry of the body and flow passage area, (iii) the thermo-physical properties of the fluid namely the density  $\rho$ , viscosity  $\mu$ , specific heat at constant pressure  $C_p$  and the thermal conductivity of the fluid  $k$  and (iv) whether the mechanism of convection is forced convection or free convection. The heat transfer coefficient for free convection will be generally lower than that for forced convection as the fluid velocities in free convection are much lower than those in forced convection. The heat transfer coefficients for some typical applications are given in table 1.2.

**Table 1.2: Typical values of the convective heat transfer coefficient  $h$**

Type of flow	$h, W / (m^2 - K)$
<b>Free convection</b>	
Gases	2 – 25
Liquids	50 – 1000
<b>Forced Convection</b>	
Gases	25 – 250
Liquids	50 – 20,000
<b>Convection with change of phase</b>	
Boiling or condensation	2500 – 100,000

**Thermal Radiation:** - Thermal radiation is the energy emitted by matter (solid, liquid or gas) by virtue of its temperature. This energy is transported by electromagnetic waves (or alternatively, photons). While the transfer of energy by conduction and convection requires the presence of a material medium, radiation does not require. In fact, radiation transfer occurs most effectively in vacuum.

Consider radiation transfer process for the surface shown in Fig.



Radiation exchange: (a) at a surface and (b) between a surface and large surroundings

Radiation that is emitted by the surface originates from the thermal energy of matter bounded by the surface, and the rate at which this energy is released per unit area is called as the surface *emissive power E*. An ideal surface is one which emits maximum emissive power and is called *an ideal radiator or a black body*. Stefan-Boltzman’s law of radiation states that the emissive power of a black body is proportional to the fourth power of the absolute temperature of the body. Therefore, if  $E_b$  is the emissive power of a black body at temperature  $T$  °K, then

$$E_b \propto T^4$$

or 
$$E_b = \sigma T^4 \dots\dots\dots(1.7)$$

Where,  $\sigma$  is the *Stefan-Boltzman constant* ( $\sigma = 5.67 \times 10^{-8} \text{ W / (m}^2 - \text{K}^4)$ ). For a non-black surface the emissive power is given by

$$E = \epsilon \sigma T^4 \dots\dots\dots(1.8)$$

where  $\epsilon$  is called the *emissivity* of the surface ( $0 \leq \epsilon \leq 1$ ). The emissivity provides a measure of how efficiently a surface emits radiation relative to a black body. The emissivity strongly depends on the surface material and finish.

Radiation may also *incident* on a surface from its surroundings. The rate at which the radiation is incident on a surface per unit area of the surface is called the *“irradiation”* of the surface and is denoted by  $G$ . The fraction of this energy absorbed by the surface is called *“absorptivity”* of the surface and is denoted by the symbol  $\alpha$ . The fraction of the incident energy is reflected and is called the *“reflectivity”* of the surface denoted by  $\rho$  and the remaining fraction of the incident energy is transmitted through the surface and

is called the *“transmissivity”* of the surface denoted by  $\tau$ . It follows from the definitions of  $\alpha$ ,  $\rho$ , and  $\tau$  that

$$\alpha + \rho + \tau = 1 \dots\dots\dots(1.9)$$

Therefore, the energy absorbed by a surface due to any radiation falling on it is given by

$$G_{abs} = \alpha G \dots\dots\dots (1.10)$$

The absorptivity  $\alpha$  of a body is generally different from its emissivity. However, in many practical applications, to simplify the analysis  $\alpha$  is assumed to be equal to its emissivity  $\epsilon$ .

**Radiation Exchange:** - Let us consider a small opaque plate (for an opaque surface  $\tau = 0$ ) of area  $A$ , emissivity  $\epsilon$  and maintained at a uniform temperature  $T_s$ . Let this plate is exposed to a large surroundings of area  $A_{su}$  ( $A_{su} \gg A$ ) which is at a uniform temperature  $T_{sur}$  as shown in Fig. The space between them contains air which is transparent to thermal radiation.

The radiation energy emitted by the plate is given by

$$Q_{em} = A \epsilon \sigma T_s^4$$

The large surroundings can be approximated as a black body in relation to the small plate. Then the radiation flux emitted by the surroundings is  $\sigma T_{sur}^4$  which is also the radiation flux incident on the plate. Therefore, the radiation energy absorbed by the plate due to emission from the surroundings is given by

$$Q_{ab} = A \alpha \sigma T_{sur}^4.$$

The net radiation loss from the plate to the surroundings is therefore given by

$$Q_{rad} = A \epsilon \sigma T_s^4 - A \alpha \sigma T_{sur}^4.$$

Assuming  $\alpha = \epsilon$  for the plate the above expression for  $Q_{net}$  reduces to

$$Q_{rad} = A \epsilon \sigma [T_s^4 - T_{sur}^4] \dots\dots\dots (1.11)$$

**Radiation Heat Transfer Coefficient:** - Under certain restrictive conditions it is possible to simplify the radiation heat transfer calculations by defining a radiation heat transfer coefficient  $h_r$  analogous to convective heat transfer coefficient as

$$Q_r = h_r A \Delta T$$

For the example of radiation exchange between a surface and the surroundings [Eq. (1. 11)] using the concept of radiation heat transfer coefficient we can write

$$Q_r = h_r A [T_s - T_{sur}] = A \epsilon \sigma [T_s^4 - T_{sur}^4]$$

$$\epsilon \sigma [T_s^4 - T_{sur}^4] \quad \epsilon \sigma [T_s^2 + T_{sur}^2][T_s + T_{sur}][T_s - T_{sur}]$$

Or 
$$h_r = \frac{\epsilon \sigma [T_s^4 - T_{sur}^4]}{[T_s - T_{sur}]} = \frac{\epsilon \sigma [T_s^2 + T_{sur}^2][T_s + T_{sur}][T_s - T_{sur}]}{[T_s - T_{sur}]}$$

Or 
$$h_r = \epsilon \sigma [T_s^2 + T_{sur}^2][T_s + T_{sur}] \dots\dots\dots (1.12)$$

**Boundary and Initial Conditions:**

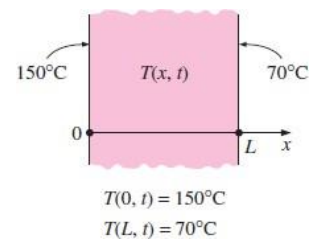
The temperature distribution within any solid is obtained by integrating the above conduction equation with respect to the space variable and with respect to time. The solution thus obtained is called the “*general solution*” involving arbitrary constants of integration. The solution to a particular conduction problem is arrived by obtaining these constants which depends on the conditions at the bounding surfaces of the solid as well as the initial condition. The thermal conditions at the boundary surfaces are called the “*boundary conditions*” . Boundary conditions normally encountered in practice are:

- (i) *Specified temperature* (also called as *boundary condition of the first kind*),
- (ii) *Specified heat flux* (also known as *boundary condition of the second kind*),
- (iii) *Convective boundary condition* (also known as *boundary condition of the third kind*)

The mathematical representations of these boundary conditions are illustrated by means of a few examples below.

- (i) **Specified Temperatures at the Boundary:** - Consider a plane wall of thickness  $L$  whose outer surfaces are maintained at temperatures  $T_0$  and  $T_L$  as shown in Fig. For one-dimensional unsteady state conduction, the boundary conditions can be written as

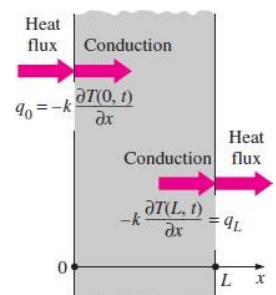
(i) at  $x = 0$ ,  $T(0,t) = T_0$  ; (ii) at  $x = L$ ,  $T(L,t) = T_L$ .



- (ii) **Specified heat flux at the boundary:** - Consider a rectangular plate as shown in Fig. and whose boundaries are subjected to the prescribed heat flux conditions as shown in the figure. Then the boundary conditions can be mathematically expressed as follows.

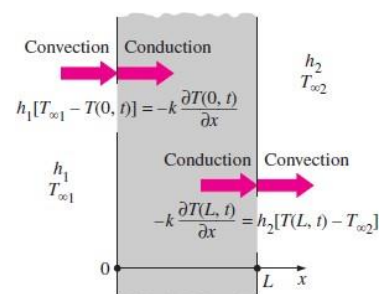
(i) at  $x = 0$ ,  $q_0 = -k (\partial T / \partial x) |_{x=0}$ ;

(ii) at  $x = L$ ,  $-k (\partial T / \partial x) |_{x=L} = q_L$ ;



- (iii) **Boundary surface subjected to convective heat transfer:** -

Fig. shows a plane wall whose outer surfaces are subjected to convective boundary conditions. The surface at  $x = 0$  is in contact with a fluid which is at a uniform temperature  $T_i$  and the surface heat transfer coefficient is  $h_i$ . Similarly, the other surface at  $x = L$  is in contact with another fluid at a uniform temperature  $T_0$  with a surface heat transfer coefficient  $h_0$ . This type of boundary condition is normally referred to as the boundary condition of third kind.



The mathematical representation of the boundary conditions for the two surfaces of the plane wall can be written as follows.

(i) at  $x = 0$ ,  $q_{\text{convection}} = q_{\text{conduction}}$ ; i.e.,  $h_i [T_i - T |_{x=0}] = -k(dT / dx) |_{x=0}$

(ii) at  $x = L$ ,  $-k(dT / dx) |_{x=L} = h_0 [T |_{x=L} - T_0]$



**General Heat Conduction Equation in Cartesian coordinate system:**

Let us consider a volume element of dimensions  $\Delta x$ ,  $\Delta y$  and  $\Delta z$  in  $x$ ,  $y$  and  $z$  directions respectively. The conduction heat transfer across the six surfaces of the element is shown in Fig.

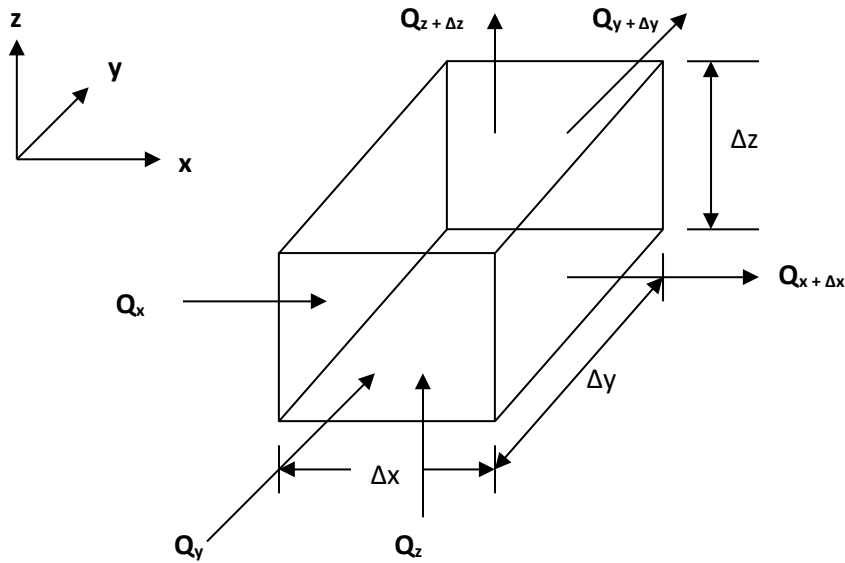


Fig. 2.3: Conduction heat transfer across the six faces of a volume element

Net Rate of conduction into the element in  $x$ -direction =  $Q_x - Q_{x+\Delta x}$

$$= Q_x - [Q_x + (\partial Q_x / \partial x) \Delta x + (\partial^2 Q_x / \partial x^2) (\Delta x)^2 / 2! + \dots]$$

$$= - (\partial Q_x / \partial x) \Delta x \text{ by neglecting higher order terms.}$$

$$= - \partial / \partial x [-k_x \Delta y \Delta z (\partial T / \partial x)] \Delta x$$

$$= \partial / \partial x [k_x (\partial T / \partial x)] \Delta x \Delta y \Delta z$$

Similarly, the net rate of conduction into the element

$$\text{in } y \text{ - direction} \quad = \partial / \partial y [k_y (\partial T / \partial y)] \Delta x \Delta y \Delta z$$

$$\text{and in } z \text{ - direction} \quad = \partial / \partial z [k_z (\partial T / \partial z)] \Delta x \Delta y \Delta z.$$

Hence the net rate of conduction into the element from all the three directions

$$Q_{in} = \{ \partial / \partial x [k_x (\partial T / \partial x)] + \partial / \partial y [k_y (\partial T / \partial y)] + \partial / \partial z [k_z (\partial T / \partial z)] \} \Delta x \Delta y \Delta z$$

$$\text{Rate of heat thermal energy generation in the element} = Q_g = q''' \Delta x \Delta y \Delta z$$

$$\text{Rate of increase of internal energy within the element} = \partial E / \partial t = \rho \Delta x \Delta y \Delta z C_p (\partial T / \partial t)$$

Applying I law of thermodynamics for the volume element we have

$$Q_{in} + Q_g = \partial E / \partial t$$

Substituting the expressions for  $Q_{in}$ ,  $Q_g$  and  $\partial E / \partial t$  and simplifying we get

$$\left\{ \frac{\partial}{\partial x} [k_x \left( \frac{\partial T}{\partial x} \right)] + \frac{\partial}{\partial y} [k_y \left( \frac{\partial T}{\partial y} \right)] + \frac{\partial}{\partial z} [k_z \left( \frac{\partial T}{\partial z} \right)] \right\} + q''' = \rho C_p \left( \frac{\partial T}{\partial t} \right)$$

The above equation is the most general form of conduction equation in Cartesian coordinate system. This equation reduces to much simpler form for many special cases as indicated below.

Special cases:-

(i) For isotropic solids, thermal conductivity is independent of direction; i.e.,  $k_x = k_y = k_z = k$ .

Hence Eq. reduces to

$$\left\{ \frac{\partial}{\partial x} [k \left( \frac{\partial T}{\partial x} \right)] + \frac{\partial}{\partial y} [k \left( \frac{\partial T}{\partial y} \right)] + \frac{\partial}{\partial z} [k \left( \frac{\partial T}{\partial z} \right)] \right\} + q''' = \rho C_p \left( \frac{\partial T}{\partial t} \right)$$

(ii) For isotropic solids with constant thermal conductivity the above equation further reduces to

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q'''}{k} = \left( \frac{1}{\alpha} \right) \left( \frac{\partial T}{\partial t} \right)$$

The above equation is called as the "Fourier – Biot equation".

(iii) Steady state conduction [i.e.,  $(\partial T / \partial t) = 0$ ]

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q'''}{k} = 0$$

The above equation is called the "Poisson equation".

(iv) No thermal energy generation [i.e.  $q''' = 0$ ]:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \left( \frac{1}{\alpha} \right) \left( \frac{\partial T}{\partial t} \right)$$

The above equation is called the "diffusion equation".

(v) Steady state conduction without heat generation [i.e.,  $(\partial T / \partial t) = 0$  and  $q''' = 0$ ]:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0$$

The above equation is called the "Laplace equation".

### Three-dimensional conduction equation in cylindrical coordinate system:

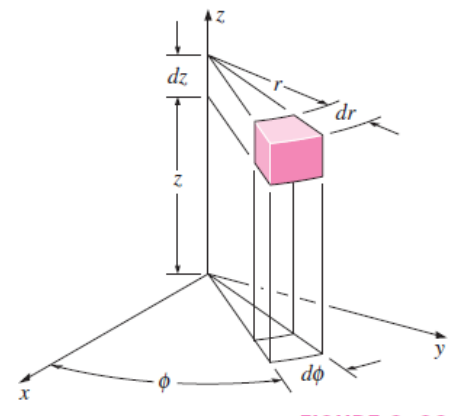
It is convenient to express the governing conduction equation in cylindrical coordinate system when we want to analyse conduction in cylinders. Any point P in space can be located by using the cylindrical coordinate system  $r$ ,  $\theta$  and  $z$  and its relation to the Cartesian coordinate system can be written as follows:

$$x = r \cos \theta ; y = r \sin \theta ; z = z.$$

Using these transformations and after laborious simplifications Eq. simplifies to

$$\frac{1}{r} \frac{\partial}{\partial r} \left( kr \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \phi} \left( kr \frac{\partial T}{\partial \phi} \right) + \frac{\partial}{\partial z} \left( k \frac{\partial T}{\partial z} \right) + g = \rho C \frac{\partial T}{\partial t}$$

The above equation is valid for only for isotropic solids.



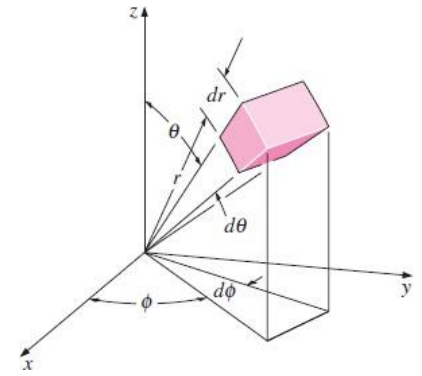
### Three-dimensional conduction equation in Spherical coordinate system:

For spherical solids, it is convenient to express the governing conduction equation in spherical coordinate system. Any point P on the surface of a sphere of radius  $r$  can be located by using the spherical coordinate system  $r$ ,  $\theta$  and  $\phi$  and its relation to the Cartesian coordinate system can be written as follows:

$$x = r \sin \phi \cos \theta ;$$

$$y = r \sin \phi \sin \theta ;$$

$$z = r \cos \phi$$

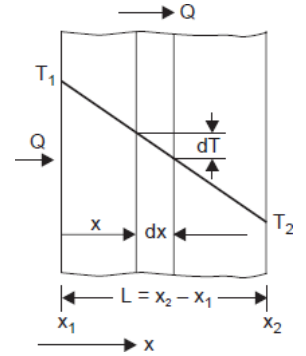


Using the relation between  $x, y, z$  and  $r, \theta$  and  $\phi$ , the conduction equation can be transformed into the equation in terms of  $r, \theta$  and  $\phi$  as follows.

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left( kr^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial}{\partial \phi} \left( k \frac{\partial T}{\partial \phi} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( k \sin \theta \frac{\partial T}{\partial \theta} \right) + \dot{g} = \rho C \frac{\partial T}{\partial t}$$

**Steady-state one-dimensional heat conduction in Cartesian System:**

**The Plane Wall (The Slab):** - The statement of the problem is to determine the temperature distribution and rate of heat transfer for one dimensional steady state conduction in a plane wall without heat generation subjected to specified boundary conditions.



The governing equation for one – dimensional steady state conduction without heat generation is given as

$$\frac{d^2T}{dx^2} = 0 \dots\dots\dots(1)$$

Integrating Eq.(1) twice with respect to x we get

$$T = C_1 x + C_2 \dots\dots\dots(2)$$

where C<sub>1</sub> and C<sub>2</sub> are constants which can be evaluated by knowing the boundary conditions.

*Plane wall with specified boundary surface temperatures:* - If the surface at x = x<sub>1</sub> is maintained at a uniform temperature T<sub>1</sub> and the surface at x = x<sub>2</sub> is maintained at another uniform temperature T<sub>2</sub>, then the boundary conditions can be written as follows:

(i) at x = x<sub>1</sub>, T(x) = T<sub>1</sub>; (ii) at x = x<sub>2</sub>, T(x) = T<sub>2</sub>.

Condition (i) in Eq. (2) gives T<sub>1</sub> = C<sub>2</sub>.

Condition (ii) in Eq. (2) gives T<sub>2</sub> = C<sub>1</sub>L + T<sub>1</sub>

or 
$$C_1 = \frac{T_2 - T_1}{L}$$

Substituting for C<sub>1</sub> and C<sub>2</sub> in Eq. (2), we get the temperature distribution in the plane wall as

$$T(x) = (T_2 - T_1) \frac{x}{L} + T_1$$

or 
$$\frac{T(x) - T_1}{(T_2 - T_1)} = \frac{x}{L} \dots\dots\dots(3)$$

*Expression for Rate of Heat Transfer:*

The rate of heat transfer at any section x is given by Fourier’s law as

$$Q_x = - k A(x) (dT / dx)$$

For a plane wall A(x) = constant = A. From Eq. (3), dT/dx = (T<sub>2</sub> – T<sub>1</sub>) / L.

Hence Q<sub>x</sub> = - k A (T<sub>2</sub> – T<sub>1</sub>) / L.

Or 
$$Q_x = \frac{kA(T_1 - T_2)}{L} \dots\dots\dots(4)$$

**Concept of thermal resistance for heat flow:**

It can be seen from the above equation that  $Q_x$  is independent of  $x$  and is a constant.

Eq.(4) can be written as

$$Q_x = \frac{(T_1 - T_2)}{\{L / (kA)\}} = \frac{(T_1 - T_2)}{R} \dots\dots\dots (5)$$

where  $R = L / (Ak)$ .

Eq. (5) is analogous to Ohm's law for flow of electric current. In this equation  $(T_1 - T_2)$  can be thought of as "thermal potential",  $R$  can be thought of as "thermal resistance", so that the plane wall can be represented by an equivalent "thermal circuit" as shown in fig.

The units of thermal resistance  $R$  is  $^{\circ}K / W$ .

**Steady State conduction in composite medium:**

There are many engineering applications in which heat transfer takes place through a medium composed of several different layers, each having different thermal conductivity. These layers may be arranged in series or in parallel or they may be arranged with combined series-parallel arrangements. Such problems can be conveniently solved using electrical analogy as illustrated in the following sections.

**Composite Plane wall: -**

**(i) Layers in series:** Consider a plane wall consisting of three layers in series with perfect thermal contact as shown in Fig. The equivalent thermal resistance network is also shown. If  $Q$  is the rate of heat transfer through an area  $A$  of the composite wall then we can write the expression for  $Q$  as follows:

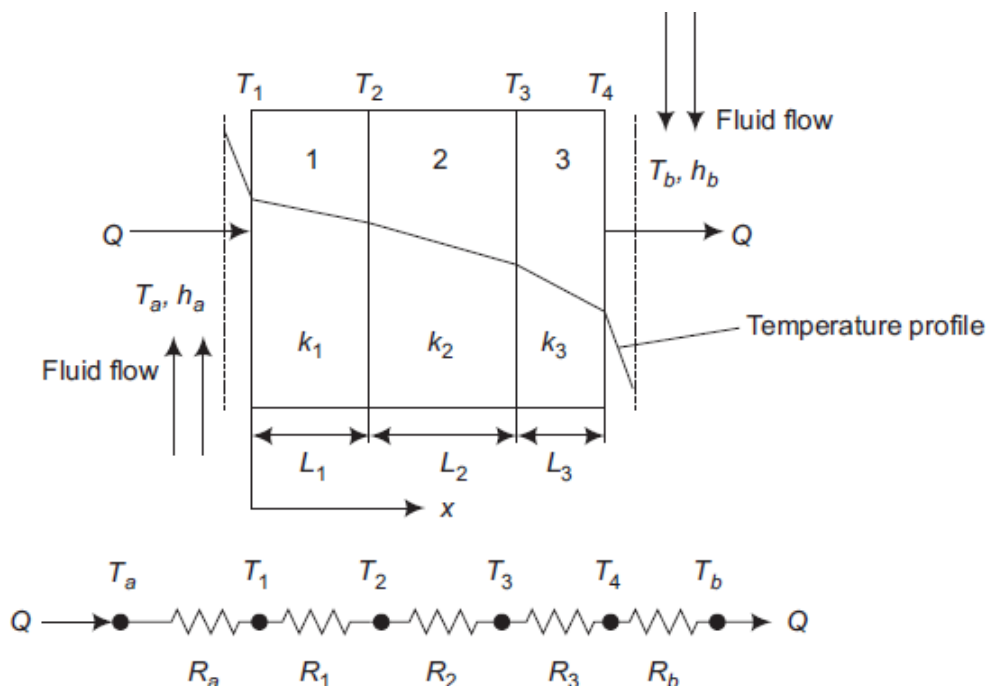


Fig.: A composite plane wall with three layers in series and the equivalent thermal resistance network

$$Q = \frac{(T_a - T_1)}{R_a} = \frac{(T_1 - T_2)}{R_1} = \frac{(T_2 - T_3)}{R_2} = \frac{(T_3 - T_b)}{R_b}$$

or 
$$Q = \frac{(T_a - T_b)}{R_a + R_1 + R_2 + R_3 + R_b} = \frac{(T_a - T_b)}{R_{total}} \dots\dots\dots(6)$$

**Overall heat transfer coefficient for a composite wall:** - It is sometimes convenient to express the rate of heat transfer through a medium in a manner which is analogous to the Newton’s law of cooling as follows:

If U is the overall heat transfer coefficient for the composite wall shown in Fig. then

$$Q = U A (T_a - T_b) \dots\dots\dots(7)$$

Comparing Eq. (6) with Eq. (7) we have the expression for U as

$$U = \frac{1}{A R_{total}} \dots\dots\dots(8)$$

or 
$$U = \frac{1}{A [ R_a + R_1 + R_2 + R_3 + R_b ]} = \frac{1}{A [ 1/(h_a A) + L_1/(A k_1) + L_2/(A k_2) + L_3/(A k_3) + 1/(h_b A) ]}$$

or 
$$U = \frac{1}{[ 1/h_a + L_1 / k_1 + L_2 / k_2 + L_3 / k_3 + 1/h_b ]} \dots\dots\dots(9)$$

**(ii) Layers in Parallel:** - Fig. shows a composite plane wall in which three layers are arranged in parallel. Let ‘b’ be the dimension of these layers measured normal to the plane of the paper. Let one surface of the composite wall be in contact with a fluid at temperature  $T_i$  and surface heat transfer coefficient  $h_i$  and the other surface of the wall be in contact with another fluid at temperature  $T_o$  with surface heat transfer coefficient  $h_o$ . The equivalent thermal circuit for the composite wall is also shown in Fig. 3.11. The rate of heat transfer through the composite wall is given by

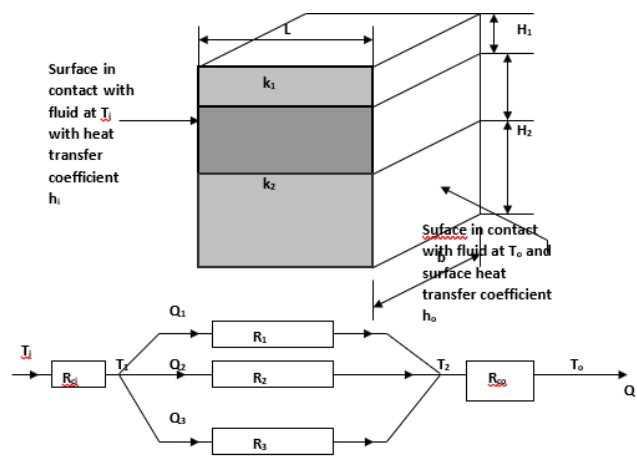


Fig.: Schematic and equivalent thermal circuit for a composite wall with layers in parallel

The rate of heat transfer through the composite wall is given by

$$Q = Q_1 + Q_2 + Q_3 \dots\dots\dots(10)$$

- where  $Q_1$  = Rate of heat transfer through layer 1,
- $Q_2$  = Rate of heat transfer through layer 2, and
- $Q_3$  = Rate of heat transfer through layer 3.

$$\text{Now } Q_1 = \frac{(T_1 - T_2)}{R_1} \dots\dots\dots (11)$$

Where  $R_1 = \{L / (H_1 b k_1)\}$

$$\text{Similarly } Q_2 = \frac{(T_1 - T_2)}{R_2} \dots\dots\dots (12)$$

Where  $R_2 = \{L / (H_2 b k_2)\}$

$$\text{and } Q_3 = \frac{(T_1 - T_2)}{R_3} = \dots\dots\dots (13)$$

Where  $R_3 = \{L / (H_3 b k_3)\}$

Substituting these expressions in Eq. (10) and simplifying we get

$$Q = \frac{(T_1 - T_2)}{R_1} + \frac{(T_1 - T_2)}{R_2} + \frac{(T_1 - T_2)}{R_3} = \frac{(T_1 - T_2)}{R_e} \dots\dots\dots (14)$$

Where  $1 / R_e = 1/R_1 + 1/R_2 + 1/R_3$

$$\text{Hence } Q = \frac{(T_i - T_1)}{R_{ci}} = \frac{(T_1 - T_2)}{R_e} = \frac{(T_2 - T_o)}{R_{co}} = \frac{(T_i - T_o)}{[R_{ci} + R_e + R_{co}]} \dots\dots\dots (15)$$

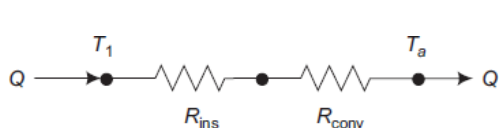
**Module 2: CRITICAL THICKNESS OF INSULATION**

For a plane wall adding more insulation will result in a decrease in heat transfer as the area of heat flow remains constant. But adding insulation to a cylindrical pipe or a conducting wire or a spherical shell will result in an increase in thermal resistance for conduction and at the same time will result in a decrease in the convection resistance of the outer surface because of increase in surface area for convection. Therefore, the heat transfer may either increase or decrease depending on the relative magnitude of these two resistances.

**Critical Radius of Insulation for Cylinder:** - Let the pipe wall be at a high temperature of  $T_1$ . Insulation is provided over this wall to reduce the heat loss to the ambient, which is at a temperature of  $T_a$ . Pipe wall loses heat to the surroundings by conduction through the insulation layer and by convection from the outer surface of insulation. So, the resistance to heat transfer is made up of two components, namely, conductive resistance through the cylindrical insulation layer  $[= \ln(r_2/r_1)/(2\pi kL)]$  and convective resistance between the wall surface and the surroundings  $[= 1/(h.A_o)]$ , where  $r_1$  is the inner radius of insulation layer (or, outer radius of pipe),  $r_2$  is the outer radius of insulation layer,  $k$  its thermal conductivity,  $L$  is length of pipe, and  $h$  is the heat transfer coefficient for convection.  $A_o$  is the area of outer surface of insulation. Obviously, as the insulation thickness is

convective resistance, given by  $[1/(h.A_o)]$  goes on decreasing since  $A_o$ , the outside surface area goes on increasing with increasing radius. Therefore, the total resistance may increase or decrease, depending on the relative rates of change of these two resistances. And, there are situations where the total resistance does decrease as the insulation thickness increases i.e. the heat transfer rate  $Q$  increases as the thickness is increased! Let us analyse when this happens.

For the above case, the equivalent thermal resistance circuit is shown below



Consider any radius  $r$  of the insulation. Let us investigate the variation of the two resistances and therefore, of  $Q$ , as the insulation radius  $r$  varies:

The total temperature potential for heat flow is  $(T_1 - T_a)$ . The resistances involved are:

- (i)  $R_{ins}$  = conductive resistance of the cylindrical insulation layer i.e.,
- (ii)  $R_{conv}$  = convective resistance on the surface of insulation, i.e. at radius  $r$ . i.e.,

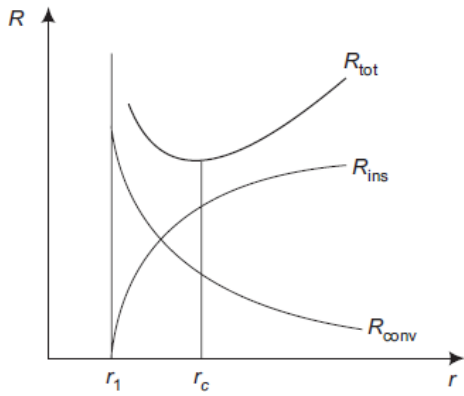
$$R_{ins} = \frac{1}{2\pi kL} \ln\left(\frac{r}{r_1}\right)$$

$$R_{conv} = \frac{1}{h_a A_o} = \frac{1}{h_a (2\pi rL)}$$

As stated earlier, conduction resistance,  $R_{ins}$  increases as  $r$  increases, and, convection resistance,  $R_{conv}$  decreases as  $r$  increases. Variation of  $R_{ins}$  and  $R_{conv}$  with  $r$  are shown in figure below; this figure also shows the variation of the total resistance,  $R_{tot}$  given by:

$$R_{tot} = R_{ins} + R_{conv}$$

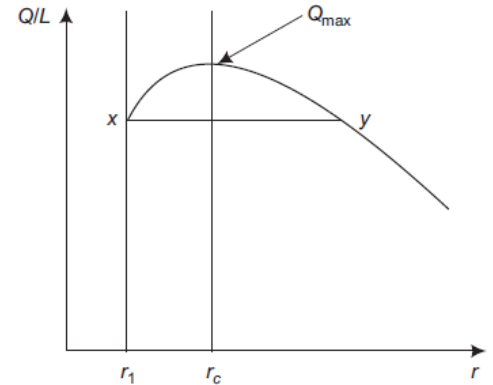




Note that  $R_{tot}$  passes through a minimum. The insulation radius at which the resistance to heat flow is minimum is called 'critical radius',  $r_c$ ; i.e. the heat flow is a maximum at the critical radius.

Correspondingly, the variation of heat flow per unit length,  $(Q/L)$ , with  $r$  is shown in figure below. The,  $r_1$  is the radius of the bare pipe and the value of  $Q/L$  at this point is the heat transfer rate per unit length for the bare pipe. Insulation is added over the pipe and till the insulation radius  $r$  reaches the value of  $r_c$ ,  $Q/L$  goes on

increasing and reaches a maximum at  $r = r_c$ . As the insulation radius is increased further,  $Q/L$  decreases, but is still at value higher than that for the bare pipe. As can be seen from the figure, at point  $y$ , value of  $Q/L$  is the same as at point  $x$ , i.e. the value of  $Q/L$  for the bare pipe. Beyond the value of  $r$  corresponding to point  $y$ , value of  $Q/L$  decreases with  $r$  and the insulation becomes really effective. It should be noted that when the radius of the pipe  $r_1$  is less than that of critical radius  $r_c$ , the insulation is not really effective in the radius range of  $r_1$  to  $r_c$ , since adding the insulation actually increases the heat flow rate.



Mathematically, to find out at what insulation radius  $r$  the  $R_{tot}$  becomes a minimum for the cylindrical system, let us differentiate the expression for  $R_{tot}$  and equate to zero; then, to confirm that at that  $r_c$ , the  $R_{tot}$  indeed goes through a minimum, verify that the second derivative of  $R_{tot}$  w.r.t.  $r$  is positive:

$$\text{Now, } R_{tot} = R_{ins} + R_{conv} = \frac{1}{2\pi kL} \ln\left(\frac{r}{r_1}\right) + \frac{1}{h_a(2\pi rL)}$$

In the above expression for  $R_{tot}$ ,  $r$  the external radius of insulation, is the only variable. So, to find out at what value of  $r$  the  $R_{tot}$  is minimum, differentiate  $R_{tot}$  w.r.t  $r$  and equate to zero:

$$\begin{aligned} \frac{d}{dr}(R_{tot}) &= \frac{1}{2\pi kL} \times \frac{r_1}{r} \times \frac{1}{r_1} + \frac{1}{h_a(2\pi L)} \times \frac{(-1)}{r^2} = 0 \\ \frac{1}{2\pi kL} \times \frac{1}{r} &= \frac{1}{h_a(2\pi L)} \times \frac{1}{r^2} \\ r &= r_c = \frac{k}{h_a} \end{aligned}$$

The above equation gives the expression for critical radius,  $r_c$  for the cylindrical system.

**Critical thickness of insulation for a sphere:** Case of a sphere is similar to that of cylinder since here also, as the radius of insulation increases, the surface area increases. So, as the insulation radius is increased, the conduction resistance of insulation increases and the convection resistance decreases.

Let  $r_1$  be the radius of the sphere on which insulation is applied, and let  $r$  be the outer radius of insulation. We would like to investigate the change of  $R_{tot}$  as insulation radius  $r$  is varied:

We have, for the spherical system:

$$R_{tot} = R_{ins} + R_{conv} = \frac{r - r_1}{4\pi k r r_1} + \frac{1}{h_a(4\pi r^2)}$$

Differentiating  $R_{tot}$  w.r.t.  $r$  and equating to zero:

$$\frac{d}{dr}(R_{tot}) = \frac{d}{dr} \left[ \frac{1}{4\pi k} \left( \frac{1}{r_1} - \frac{1}{r} \right) + \frac{1}{h_a(4\pi r^2)} \right] = 0$$

$$\text{i.e. } \frac{1}{4\pi k} \left( 0 + \frac{1}{r^2} \right) - \frac{2}{h_a 4\pi r^3} = 0$$

$$\text{i.e. } \frac{1}{4\pi k r^2} - \frac{2}{h_a 4\pi r^3} = 0$$

$$\text{i.e. } r = r_c = \frac{2k}{h_a}$$

The above equation gives the expression for critical radius,  $r_c$  for the spherical system.

**EXTENDED SURFACES (FINS)****Introduction:**

Convection: Heat transfer between a solid surface and a moving fluid is governed by the Newton's cooling law:  $Q = hA(T_s - T_\infty)$ , where  $T_s$  is the surface temperature and  $T_\infty$  is the fluid temperature. Therefore, to increase the convective heat transfer, one can

- Increase the temperature difference ( $T_s - T_\infty$ ) between the surface and the fluid.
- Increase the convection coefficient  $h$ . This can be accomplished by increasing the fluid flow over the surface since  $h$  is a function of the flow velocity and the higher the velocity, the higher the  $h$ . Example: a cooling fan.
- Increase the contact surface area  $A$ . Example: a heat sink with fins.

Many times, when the first option is not in our control and the second option (i.e. increasing  $h$ ) is already stretched to its limit, we are left with the only alternative of increasing the effective surface area by using fins or extended surfaces. Fins are protrusions from the base surface into the cooling fluid, so that the extra surface of the protrusions is also in contact with the fluid. Most of you have encountered cooling fins on air-cooled engines (motorcycles, portable generators, etc.), electronic equipment (CPUs), automobile radiators, air conditioning equipment (condensers) and elsewhere.

Fins are generally classified as (i) straight fins, (ii) annular fins and (iii) spines.

The term straight fin is applied to the extended surface attached to a wall which is otherwise plane. The fin cross sectional area may or may not vary along the length of the fin. An annular fin is one which is attached, circumferentially, to a cylindrical surface. A spine or pin fin is an extended surface of cylindrical or conical shape.

**Extended surface analysis:** In this section consideration will be limited to steady state analysis of rectangular or pin fins of constant cross-sectional area. Annular fins or fins involving a tapered cross section may be analysed by similar methods, but will involve solution of more complicated equations. Numerical methods of integration or computer programs can be used to advantage in such cases.

**One dimensional approximation for extended surface analysis:** We can examine when temperature variation across the fin can be neglected in comparison to the temperature variation along the length of the fin. Consider the heat transfer in  $y$  direction for the fin of length  $L$ , width  $W$  and uniform thickness  $t$  ( $t \ll L$  and  $t \ll W$ ) as shown in figure. At any  $x$ , if the rate of heat transfer in  $x$  direction is of the same order of magnitude as the rate in  $y$  direction, we have

$$-k A_x \frac{\partial \theta}{\partial x} \approx -k A_y \frac{\partial \theta}{\partial y}$$

Using first order linear approximation, the above equation can be written as

$$-k t W \frac{(\Delta T)_x}{L} \approx -k L W \frac{(\Delta T)_y}{\frac{t}{2}}$$

or

$$\frac{(\Delta T)}{(\Delta T)_x} \approx \frac{t^2}{L^2}$$

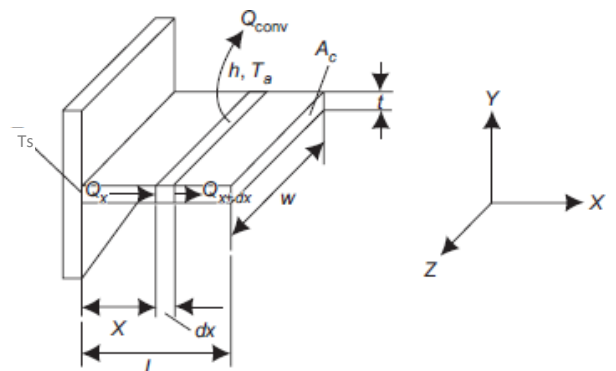
For fins  $t$  is very much smaller than  $L$ ; therefore, for the heat transfers to be of the same order of magnitude, it follows that the temperature variation in  $y$  direction must be much smaller than the temperature variation in  $x$  direction: i.e.  $\frac{\partial^2 T}{\partial y^2} \ll \frac{\partial^2 T}{\partial x^2}$ . Hence conduction problems in fins is treated as one-dimensional where temperature varies along the length of the fin.

**Governing differential equation for 1-D analysis:** The one-dimensional steady state Conduction Equation in solids with heat generation and constant thermal conductivity is given by:

$$\frac{d^2 T}{dx^2} + \frac{q'''}{k} = 0 \text{ ----- (a)}$$

This is a second order, ordinary differential equation and will require 2 boundary conditions to evaluate the two constants of integration that will arise.

Consider the cooling fin shown in figure below. The fin is situated on the surface of a hot surface at  $T_s$  and surrounded by a coolant at temperature  $T_a$ , which cools with convective coefficient,  $h$ . The fin has a uniform cross-sectional area,  $A_c$ , (This is the area through with heat is conducted.) and an overall length,  $L$ . Note that as energy is conducted down the length of the fin, some portion is lost, by convection, from the sides. Thus, the heat flow varies along the length of the fin. We further note that the arrows indicating the direction of heat flow point in both the  $x$  and  $y$  directions. This is an indication that this is truly a two- or three-dimensional heat flow, depending on the geometry of the fin. However, quite often, it is convenient to analyse a fin by examining an equivalent one-dimensional system. The equivalent system will involve the introduction of heat sinks (negative heat sources), which remove an amount of energy equivalent to what would be lost through the sides by convection.



Consider a differential length of the fin. Across this segment the heat loss will be  $h(P\Delta x)(T - T_\infty)$ , where  $P$  is the perimeter around the fin. The equivalent heat sink would be  $-q''' A_c \Delta x$ . Equating the heat source to the convective loss we get

$$-q''' A_c \Delta x = hP\Delta x (T - T_\infty)$$

or:

$$q''' = -\frac{hP}{kA_c} (T - T_\infty)$$

Substituting this expression for  $q'''$  into Equation (a) we get

$$\frac{d^2 T}{dx^2} - \frac{hP}{kA_c} (T - T_\infty) = 0.$$

or:

$$\frac{d^2 T}{dx^2} - m^2 (T - T_\infty) = 0. \text{ ----- (b)}$$

Where

$$m^2 = \frac{hP}{kA_c}$$

which is the equation for a fin with a constant cross-sectional area. This is a Second Order linear differential Equation.

**Solution to the fin equation:** We notice that equation for  $q'''$  is non-homogeneous (due to the  $T_\infty$  term). Recall that non-homogeneous differential equations require both a homogeneous and a particular solution. We can make this equation homogeneous by introducing the fin temperature relative to the surroundings:

$$\text{Let } \theta \equiv T - T_\infty$$

Then Equation (b) reduces to,

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0 \text{----- (c)}$$

The above equation is called the **fin equation**.

The above equation is a second order linear homogeneous differential equation the general solution for which is given by

$$\theta = Ae^{mx} + Be^{-mx}$$

The solution for Equation (c) is also given by

$$\theta = C \cosh mx + D \sinh mx$$

Generally, the exponential solution is used for very long fins, the hyperbolic solutions for other cases to facilitate easy evaluation of constants of integration.

**Boundary Conditions:** Since the solution results in 2 constants of integration we require 2 boundary conditions. The first one is obvious, as one end of the fin will be attached to a hot surface and will come into thermal equilibrium with that surface.

Hence, at the fin base ( $x = 0$ ),

$$\theta = T_0 - T_\infty = \theta_0$$

The second boundary condition depends on the condition imposed at the other end of the fin. There are various possibilities, as described below.

*(i) Very long fins:* For very long fins, the end located a long distance from the heat source will approach the temperature of the surroundings. Hence,

$$\theta(\infty) = 0$$

Substitute the second condition into the exponential solution of the fin equation:

$$\theta(\infty) = 0 = Ae^{m\infty} + Be^{-m\infty}$$

The first exponential term is infinite and the second is equal to zero. The only way that this equation can be satisfied is if  $A = 0$ . Now apply the second boundary condition.

$$\theta(0) = \theta_0 = B$$

The general temperature profile for a very long fin is then given by:

$$\theta(x) = \theta_0 e^{-mx} \text{----- (d)}$$

If we wish to find the heat flow through the fin, we may apply Fourier Law:

At any  $x$ , the rate of heat conduction is given by

$$Q = -kA_c \frac{dT}{dx} = -kA_c \frac{d\theta}{dx}$$

All the heat dissipated by the fin to the surroundings is coming from the fin base. Hence the rate of heat transfer from the fin to the surroundings is given by

$$Q_{fin} = Q|_{x=0} = -kA_c \frac{d\theta}{dx} \Big|_{x=0}$$

From Equation (3.57) we have,  $\frac{d\theta}{dx} \Big|_{x=0} = -m\theta_0$

Substituting this expression in Equation (d) we get

$$Q_{fin} = kmA_c\theta_0 = (hpkA_c \theta_0)$$

**(ii) The fin with insulated tip:** Assume that the tip is insulated and hence there is no heat transfer at the fin tip.

Hence the boundary condition at the tip is given by

At  $x = L$ ,  $\frac{d\theta}{dx} \Big|_{x=L} = 0$

Substituting this condition and the boundary condition at  $x = 0$  and simplifying we get the temperature profile for this case as

$$\theta(x) = \theta_0 \frac{\cosh m(L-x)}{\cosh mL} \text{----- (e)}$$

We may find the heat flow at any value of  $x$  by differentiating the temperature profile and substituting it into the Fourier Law:

$$Q_{fin} = Q|_{x=0} = -kA_c \frac{d\theta}{dx} \Big|_{x=0}$$

From Equation (e),  $\frac{d\theta}{dx} \Big|_{x=0} = -\theta_0 \frac{\sinh mL}{\cosh mL}$

Therefore:  $Q_{fin} = kmA_c \tanh(mL)$ ----- (f)

If we compare this result with that for the very long fin, we see that the primary difference in form is in the hyperbolic tangent term. That term, which always results in a number equal to or less than one, represents the reduced heat loss due to the shortening of the fin

**Other tip conditions:** We have already seen two tip conditions, one being the long fin and the other being the insulated tip. Two other possibilities are usually considered for fin analysis: (i) a tip subjected to convective heat transfer, and (ii) a tip with a prescribed temperature. The expressions for temperature distribution and fin heat transfer for all the four cases are summarized in the table below.

**Expressions for Temperature distribution and rate of heat dissipation from a fin for different tip conditions**

Tip Condition	Expression for Temperature Profile	Expression for Rate of Heat Transfer
Long Fin	$\theta(x) = \theta_0 e^{-mx}$	$Q_{fin} = kmA_c\theta_0$
Tip Insulated	$\theta(x) = \theta_0 \frac{\cosh m(L-x)}{\cosh mL}$	$Q_{fin} = \sqrt{hPkA_c} \theta_0 \tanh mL$
Tip with prescribed Temperature, $T_L$	$\theta(x) = \frac{\theta_0 \sinh mx + \sinh m(L-x)}{\sinh mL}$	$Q_{fin} = \sqrt{hPkA_c} \theta_0 \frac{\cosh mL - \frac{\theta_L}{\theta_0}}{\sinh mL}$
Tip dissipating heat by convection with surface heat transfer coefficient, $h_c$	$\theta(x) = \frac{\cosh m(L-x) + \frac{h_c}{km} \sinh m(L-x)}{\cosh mL + \frac{h_c}{km} \sinh mL}$	$Q_{fin} = \theta_0 \sqrt{hPkA_c} \frac{\sinh mL + \frac{h_c}{km} \cosh mL}{\cosh mL + \frac{h_c}{km} \sinh mL}$

**Performance of a fin:**

**Fin Efficiency:** The performance of a fin is assessed by using a parameter called "efficiency of a fin,  $\eta$ " given by,

$$\eta = \frac{\text{Actual rate of heat transfer from the fin}}{\text{Maximum Possible heat transfer rate from fin}} = \frac{Q_{fin}}{(Q_{fin})_{max}}$$

The rate of heat transfer from the fin will be maximum when the entire fin surface is at the fin base temperature (This means that the fin is having an infinite thermal conductivity): i.e.,

$$(Q_{fin})_{max} = hPL(T_0 - T_\infty) = hPL\theta_0$$

**Fin Effectiveness:** Fins are used to enhance the heat transfer, and the use of fins on a surface cannot be recommended unless the enhancement in heat transfer justifies the added cost and complexity associated with the fins. In fact, there is no assurance that by adding fins on a surface, the heat transfer rate will enhance. The performance of the fin is judged on the basis of enhancement in heat transfer relative to the no-fin case. To know how effective an extended surface as compared to a bare primary surface is, another parameter called effectiveness is used. It is given by,

$$\epsilon = \frac{\text{Heat Transfer with fin}}{\text{Heat Transfer without fin}} = \frac{Q_{fin}}{Q_{bare}} = \frac{Q_{fin}}{hA_b\theta_0}$$

In equation (3.64),  $A_b$  is the cross-sectional area of the fin at its base.

It should be noted that both the fin efficiency and the fin effectiveness are related to the performance of the fin, but they are different quantities. However, they are related to each other as shown below.

$$\epsilon = \frac{Q_{fin}}{hA_b\theta_0} = \frac{\eta hA_f\theta_0}{hA_b\theta_0} = \frac{A_f}{A_b} \eta \theta_0 \quad \text{----- (g)}$$

Hence, the fin effectiveness can be determined easily when the efficiency of the fin is known, or vice-versa.

**Considerations in the design and selection of fins:**

**(i) Long fin:** - The rate of heat transfer from a *very long fin of uniform cross section* under steady state conditions is given by Equation (f). Substituting this expression in Equation (g) we get,

$$\epsilon_{\text{long}} = \frac{Q_{\text{fin}}}{hA_b\theta_0} = \frac{kmA_b\theta_0}{hA_b\theta_0} = \frac{k}{h} \times \frac{hP}{kA}$$

or: 
$$\epsilon_{\text{long}} = \frac{kP}{hA} \tag{h}$$

Several important conclusions can be drawn from Equation (3.66) which helps in design and selection of long fins for various applications.

- (i) The thermal conductivity  $k$  of the fin material should be as high as possible. Hence fins are made of materials like copper, Aluminium and iron. The most widely used material is aluminium because of its low cost and weight and its resistance to corrosion.
- (ii) The ratio of the *perimeter* to the *cross-sectional area* of the fin  $P/A$  should be as high as possible. This criterion is satisfied by employing *thin* plate fins and *slender* pin fins.
- (iii) The use of fins is *most efficient* in applications involving *low convection heat transfer coefficients*. Thus, the use of fins is easily justified when the medium is *gas* instead of a liquid and the heat transfer mechanism is by *free convection* rather than by forced convection. Therefore, in a liquid to gas heat exchangers like automobile radiators, fins are placed on the gas side of the heat exchangers.

**(ii) Straight Fin of finite length:** Consider a straight fin of uniform thickness 'w' and length L. For unit width of the fin, the parameter m is given by,

$$m = \sqrt{\frac{2h}{kw}}$$

The heat dissipation for a fin of finite length is

$$Q_{\text{fin}} = \theta_0 \sqrt{hPkA_c} \frac{\sinh mL + \frac{h_e}{km} \cosh mL}{\cosh mL + \frac{h_e}{km} \sinh mL}$$

Per unit width, assuming  $h_e = h$ , the above equation reduces to

$$Q_{\text{fin}}^* = \theta_0 \sqrt{2hkw} \frac{\sinh \frac{2h}{kw} L + \frac{hw}{2k} \cosh \frac{2h}{kw} L}{\cosh \frac{2h}{kw} L + \frac{hw}{2k} \sinh \frac{2h}{kw} L}$$

The above equation can be written as:

$$Q_{\text{fin}}^* = \theta_0 \sqrt{2hkw} \frac{\tanh \frac{2h}{kw} + \frac{hw}{2k}}{1 + \frac{hw}{2k} \tanh \frac{2h}{kw} L}$$

Equation will indicate the effect of varying the length L of the fin on heat dissipation, if other things are being held fixed. Hence this equation is useful in finding out the condition under which fins must be employed to increase the heat dissipation. The condition for optimum value of  $Q_{\text{fin}}^*$  can be obtained by differentiating  $Q_{\text{fin}}^*$  with respect to L and equating it to zero.

$$\frac{dQ_{\text{fin}}^*}{dL} = \theta_0 \sqrt{2hkw} \frac{\left[ \frac{hw}{2k} \tanh \frac{2h}{kw} L + \frac{hw}{2k} \right] \frac{d}{dL} \left[ \frac{hw}{2k} \tanh \frac{2h}{kw} L \right] - \left[ \frac{hw}{2k} \tanh \frac{2h}{kw} L + \frac{hw}{2k} \right]^2 \frac{d}{dL} \left[ \frac{2h}{kw} \right] \text{sech}^2 \frac{2h}{kw} L}{2} = 0$$

or: 
$$\frac{hw}{2k} = 1$$



Further regardless of the value of  $L$ , if  $\frac{hw}{2k} > 1$ ,  $\frac{d \dot{Q}_{fin}}{dL}$  is negative. These statements are valid for all values of  $L > 0$  and hence they may be applied to determine whether or not a fin should be used at all. If  $\frac{hw}{2k} > 1$ , fins produce insulating effect. From cost considerations, fins are to be provided only if,

$$\frac{hw}{2k} \ll 1$$

**Module 2: TRANSIENT [UNSTEADY-STATE] HEAT CONDUCTION**

In second half of module 1, we derived the general differential equation for conduction and then applied it to problems of increasing complexity, e.g. first, we studied heat transfer in simple geometries without heat generation and then we studied heat transfer when there was internal heat generation. In all these problems, steady state heat transfer was assumed, i.e. the temperature within the solid was only a function of position and did not depend on time, i.e. mathematically,  $T = T(x, y, z)$ . However, all the process equipments used in engineering practice, such as boilers, heat exchangers, regenerators, etc. have to pass through an unsteady state in the beginning when the process is started, and, they reach a steady state after sufficient time has elapsed. Conduction heat transfer in such an unsteady state is known as transient heat conduction or, unsteady state conduction, or time dependent conduction.

Obviously, in transient conduction, temperature depends not only on position in the solid, but also on time. So, mathematically, this can be written as  $T = T(x, y, z, \tau)$ , where  $\tau$  represents the time coordinate.

Naturally, solutions for transient conduction problems are a little more complicated compared to steady state analysis, since now, an additional parameter, namely time ( $\tau$ ) is involved.

Typical examples of transient conduction occur in:

- a) Heat exchangers
- b) Boiler tubes
- c) Cooling of cylinder heads in I.C. engines
- d) Heat treatment of engineering components and quenching of ingots
- e) Heating of electric irons
- f) Heating and cooling of buildings
- g) Freezing of foods, etc.

Two types of transient conduction may be identified:

(a) periodic heat flow problems, where the temperatures vary on a regular, periodic basis, e.g. in I.C. engine cylinders, alternate heating and cooling of earth during a 24 hr cycle (by sun) etc.

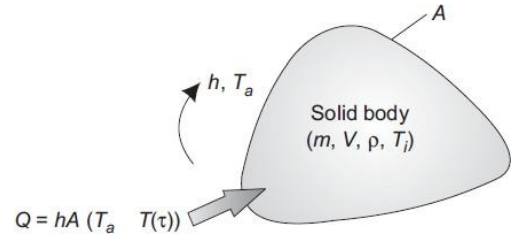
(b) non-periodic heat flow problems, where temperature varies in a non-linear manner with time.

When the temperature gradients within the solid are very small, (i.e. the internal resistance to conduction is negligible) which can be solved simply by applying the energy balance principle. Consider for example, a small body made of, say, copper, at a high temperature, being quenched in a medium like oil. Then, the body loses heat to the medium. Heat flows by conduction from within the body to the surface and then, by convection to the medium. When the body is very small or when the thermal conductivity of the material of the body is very large, temperature gradients within the body will be very small and may be neglected. In such a case, temperature within the body is only a function of time and is independent of spatial coordinates, i.e. the whole-body acts as lump and temperatures of all points within the body decrease (or increase if the object is being heated) uniformly en-mass. Heat transfer process from the body, in this case, is controlled by the convection resistance at the surface rather than by the conduction resistance in the solid. Such an analysis, where the internal resistance of the body for heat conduction is negligible and the whole

body may be treated as a lump as far as temperature increase or decrease is concerned, is known as lumped system analysis.

**Lumped System Analysis (Newtonian Heating or Cooling):** As mentioned above, in lumped system analysis, the internal conduction resistance of the body to heat flow (i.e.  $L/(k.A)$ ) is negligible compared to the convective resistance (i.e.  $1/(h.A)$ ) at the surface. So, the temperature of the body, no doubt, varies with time, but at any given instant, the temperature within the body is uniform and is independent of position, i.e.  $T = T(\tau)$  only.

Consider a solid body of arbitrary shape, volume  $V$ , mass  $m$ , density  $\rho$ , surface area  $A$ , and specific heat  $C_p$ . To start with, at  $\tau = 0$ , let the temperature throughout the body be uniform at  $T = T_i$ . At the instant  $t = 0$ , let the body be suddenly placed in a medium at a temperature of  $T_a$ , as shown. For the sake of analysis, let us assume that  $T_a > T_i$ ; however, same analysis is valid for  $T_a < T_i$  too. Then, heat will be transferred from the medium to the body and the temperature of the body will increase with time. Let the temperature of the body rise by a differential amount  $dT$  in a differential time interval  $dt$ , thus increasing the internal energy of the solid.



Writing an energy balance for this situation:

Amount of heat transferred into the body in time interval  $d\tau$  = Increase in the internal energy of the body in time interval  $d\tau$

$$h \cdot A \cdot (T_a - T(\tau)) \cdot d\tau = m \cdot C_p \cdot dT = \rho \cdot C_p \cdot V \cdot dT$$

$$m = \rho \cdot V$$

Now, since  $T_a$  is a constant, we can write:

$$dT = d(T(\tau) - T_a)$$

Therefore,

$$\frac{d(T(\tau) - T_a)}{T(\tau) - T_a} = \frac{-h \cdot A}{\rho \cdot C_p \cdot V} \cdot d\tau$$

Integrating between  $\tau = 0$  (i.e.  $T = T_i$ ) and any  $\tau$ , (i.e.  $T = T(\tau)$ ),

$$\ln\left(\frac{T(\tau) - T_a}{T_i - T_a}\right) = \frac{-h \cdot A \cdot \tau}{\rho \cdot C_p \cdot V}$$

$$\frac{T(\tau) - T_a}{T_i - T_a} = \exp\left(\frac{-h \cdot A \cdot \tau}{\rho \cdot C_p \cdot V}\right)$$

Now, let:

$$\frac{\rho \cdot C_p \cdot V}{h \cdot A} = t$$

where,  $t$  is known as thermal time constant and has units of time.

Therefore, above equation is written as:

$$\frac{T(\tau) - T_a}{T_i - T_a} = \exp\left(\frac{-\tau}{t}\right)$$

Now denoting  $\theta = (T(\tau) - T_a)$ , we write above equation compactly as:

$$\frac{\theta}{\theta_i} = \frac{T(\tau) - T_a}{T_i - T_a} = \exp\left(\frac{-\tau}{t}\right)$$

The above equation gives the temperature distribution in a solid as a function of time, when the internal resistance of the solid for conduction is negligible compared to the convective resistance at its surface.

The above equation is represented graphically in the following figure (a) and (b).

From the equation of temperature distribution, we note:

- (i) Temperature distribution is exponential, i.e. temperature changes rapidly initially and approaches that of the medium exponentially (figure (a)).
- (ii) Either the time required by the body to reach a certain temperature or the temperature attained by the body after a certain time interval, can be found out from temperature distribution equation.
- (iii) Larger the value of time constant  $t$ , longer is the time required for the body to reach a particular temperature.
- (iv) time required for the body to attain 36.8% of the applied temperature difference is indicated in the figure (b). This is known as one time period and is of importance in connection with measurement of temperatures with thermocouples. Larger the value of time constant, larger is the time period.

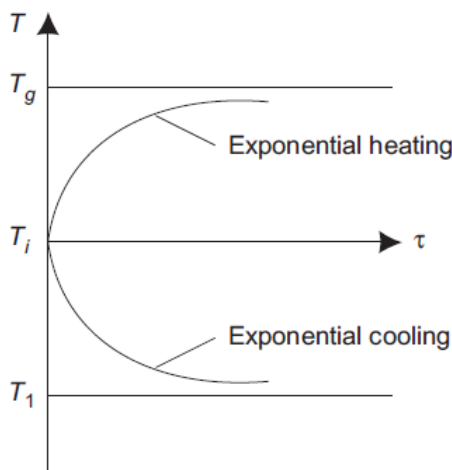


Figure (a): Newtonian heating and cooling

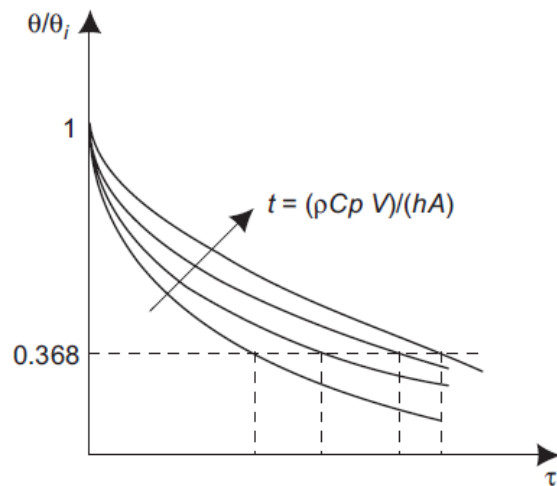


Figure (b): Temperature variation with time in a lumped system

**Instantaneous heat transfer:** At any instant  $\tau$ , heat transfer between the body and the environment is easily calculated since we have the temperature from the temperature distribution equation:

$$Q(\tau) = m \cdot C_p \cdot \frac{dT(\tau)}{d\tau}, \text{ W}$$

At that instant, heat transfer must also be equal to:

$$Q(\tau) = h \cdot A \cdot (T(\tau) - T_a), \text{ W}$$

**Total heat transfer:** Total heat transferred during  $\tau = 0$  to  $\tau = \tau$ , is equal to the change in internal energy of the body:

$$Q_{tot} = m \cdot C_p \cdot (T(\tau) - T_i), \text{ J}$$

$Q_{tot}$  may also be calculated by integrating instantaneous heat transfer equation:

$$Q_{tot} = \int_0^\tau Q(\tau) d\tau, \text{ J}$$

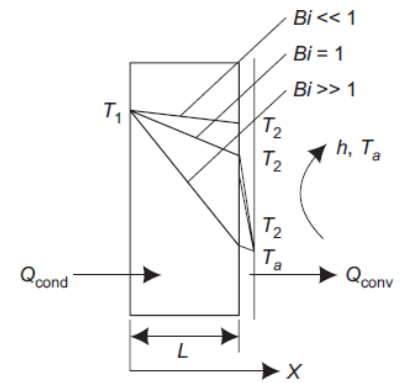
**Maximum heat transferred:** When the body reaches the temperature of the environment, obviously, maximum heat has been transferred:

$$Q_{max} = m \cdot C_p \cdot (T_a - T_i), \text{ J}$$

If  $Q_{max}$  is negative, it means that the body has lost heat, and if  $Q_{max}$  is positive, then body has gained heat.

**Criteria for Lumped System Analysis (Biot Number and Fourier Number):**

Consider a plane slab in steady state, transferring heat to a fluid on its surface with a heat transfer coefficient of  $h$ , as shown in figure. (The criterion arrived at is readily extended to transient conditions later.) Let the surface on the left be maintained at temperature  $T_1$  and the surface on the right is at a temperature of  $T_2$  as a result of heat being lost to a fluid at temperature  $T_a$ , flowing with a heat transfer coefficient  $h$ . Writing an energy balance at the right-hand surface,



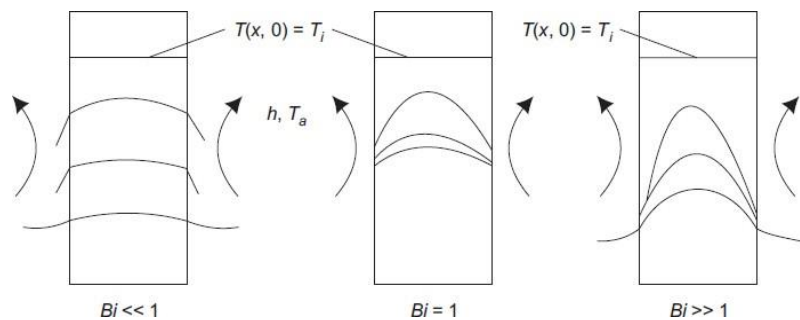
$$Q_{conduction} = Q_{convection}$$

$$\frac{kA}{L}(T_1 - T_2) = hA(T_2 - T_a)$$

Rearranging,

$$\frac{T_1 - T_2}{T_2 - T_a} = \frac{kL}{hA} = \frac{R_{cond}}{R_{conv}} = \frac{hL}{k} = Bi$$

The term,  $(h \cdot L)/k$ , appearing on the RHS of the above equation is a dimensionless number, known as **Biot number**. Biot number is a measure of the temperature drop in the solid relative to the temperature drop in the convective layer. It is also interpreted as the ratio of conductive resistance in the solid to the convective resistance at its surface. This is precisely the criterion we are looking for. Note from the above figure the temperature profile for  $Bi \ll 1$ . It suggests that one can assume a uniform temperature distribution within the solid if  $Bi \ll 1$ .



Situation during transient conduction is shown in the above figure. It may be observed that temperature distribution is a strong function of Biot number. For  $Bi \ll 1$ , temperature gradient in the solid is small and temperature can be taken as a function of time only. Note also that for  $Bi \gg 1$ , temperature drop across the solid is much larger than that across the convective layer at the surface.

Therefore, to fix the criterion for which lumped system analysis is applicable, let us define Biot number, in general, as follows:

$$Bi = \frac{hL_c}{k}$$

where,  $h$  is the heat transfer coefficient between the solid surface and the surroundings,  $k$  is the thermal conductivity of the solid, and  $L_c$  is a characteristic length defined as the ratio of the volume of the body to its surface area, i.e.

$$Characteristic\ length, L_c = \frac{V}{A}$$

With this definition of  $Bi$  and  $L_c$ , for solids such as a plane slab, long cylinder and sphere, it is found that transient temperature distribution within the solid at any instant is uniform, with the error being less than about 5%, if the following criterion is satisfied:

$$Bi \cdot L_c = \frac{hL_c}{k} < 0.1$$

In other words, if the conduction resistance of the body is less than 10% of the convective resistance at its surface, the temperature distribution within the body will be uniform within an error of 5%, during transient conditions.

**$L_c$  for common shapes:**

- 1. Plane wall (thickness  $2L$ ):  $L_c = \frac{A \cdot L}{2A} = L = \text{half the thickness of the wall}$
- 2. Long cylinder, radius  $R$ :  $L_c = \frac{\pi R^2 L}{2\pi R L} = \frac{R}{2}$
- 3. Sphere, radius,  $R$ :  $L_c = \frac{\frac{4}{3}\pi R^3}{4\pi R^2} = \frac{R}{3}$
- 4. Cube, side  $L$ :  $L_c = \frac{L^3}{6L^2} = \frac{L}{6}$

Therefore, we can write,

$$\frac{\theta}{\theta_0} = \frac{T_c - T_a}{T_\infty - T_a} = e^{-\frac{Bi}{L_c} \cdot \frac{L_c}{t}} \quad \text{for } Bi \cdot L_c < 0.1$$

**Response Time of a Thermocouple:** Lumped system analysis is usefully applied in the case of temperature measurement with a thermometer or a thermocouple. Obviously, it is desirable that the thermocouple indicates the source temperature as fast as possible. If the thermocouple is measuring changing temperatures, then also, it should follow the temperature changes at a rate faster than the rate of temperature change. 'Response time' of a thermocouple is defined as the time taken by it to reach the source temperature.

$$\frac{\theta}{\theta_0} = \frac{T_c - T_a}{T_\infty - T_a} = e^{-\frac{hAc}{\rho C_p V} t} \quad \text{for } Bi \cdot L_c < 0.1$$

For rapid response, the term  $(hAt)/(rCpV)$  should be large so that the exponential term will reach zero faster. This means that:

- (i) Increase  $(A/V)$ , i.e. decrease the wire diameter.
- (ii) Decrease density and specific heat.
- (iii) Increase the value of heat transfer coefficient  $h$ .

As mentioned earlier, the quantity  $(\rho C_p V)/(h \times A)$  is known as 'thermal time constant',  $t$ , of the measuring system and has units of time. At  $\tau = t$ , i.e. at a time interval of one time constant, we have:

$$\frac{T_c - T_a}{T_\infty - T_a} = e^{-1} = 0.368$$

From the above equation, it is clear that after an interval of time equal to one time constant of the given temperature measuring system, the temperature difference between the body (thermocouple) and the source would be 36.8% of the initial temperature difference, i.e. the temperature difference would be reduced by 63.2%.

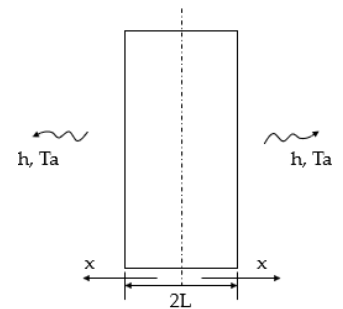
Time required by a thermocouple to attain 63.2% of the value of initial temperature difference is called its **sensitivity**.

For good response, obviously, the response time of thermocouple should be low. As a thumb rule, it is recommended that while using a thermocouple to measure temperatures, reading of the thermocouple should be taken after a time equal to about four time periods has elapsed.

Heisler’s Charts

If the Biot number is greater than 0.1, the lumped system analysis can not be applied. In such cases, the Heisler’s charts are used to find the temperature at the mid planes and at any distance from the mid plane ( $T_{(x/L)}$ ) or at any radius ( $T_{(r/R)}$ ).

Consider a plane slab of thickness  $2L$ , shown in figure. Initially, i.e. at  $\tau = 0$ , the slab is at a uniform temperature,  $T_i$ . Suddenly, at  $\tau = 0$ , both the surfaces of the slab are subjected to convection heat transfer with an ambient at temperature  $T_a$ , with a heat transfer coefficient of  $h$ , as shown. Since there is geometrical and thermal symmetry, we need to consider only half the slab, and that is the reason why we chose the origin of the coordinate system on the mid-plane. Then, we can write the mathematical formulation of the problem for plane slab as follows:



Transient heat conduction in an infinite plane wall

$$\frac{d^2T}{dx^2} = \frac{1}{\alpha} \frac{dT}{d\tau} \text{ in } 0 < x < L \text{ for } \tau > 0$$

The boundary conditions are:

- i) At  $\tau = 0$ , in  $0 < x < L$  :  $T = T_i$
- ii) At  $x = 0$ , for  $\tau > 0$  :  $\frac{dT}{dx} = 0$
- iii) At  $x = L$ , for  $\tau > 0$  :  $-k \frac{dT}{dx} = h(T - T_a)$

The solution of the above problem consists of **infinite series**. So, it is more convenient to present the solution either in **tabular form** or **charts**.

It is absorbed that is the dimensionless time,  $Fo > 0.2$ , considering only the first term of the series results in an error of less than 2% and this method is called **One Term Approximation**

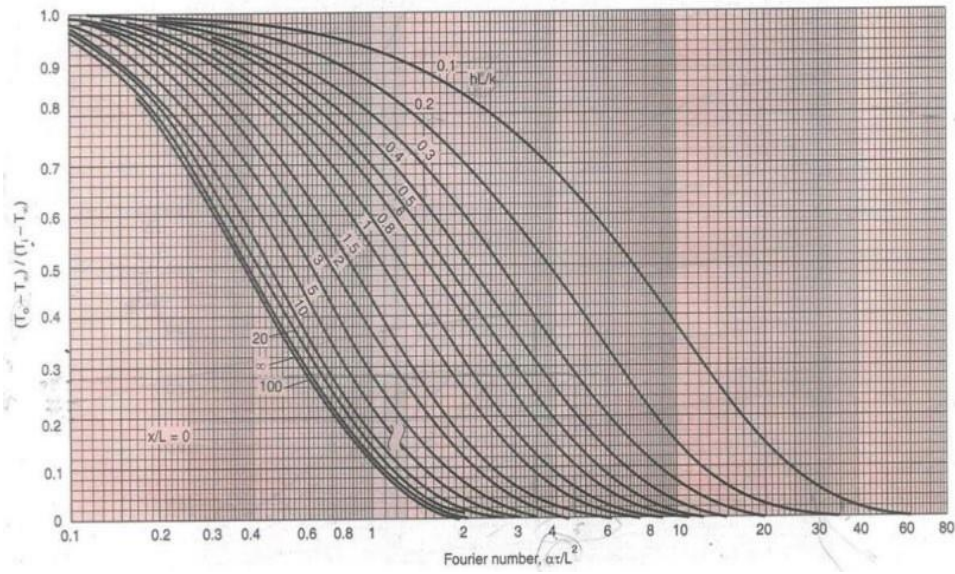
**Solution.**

1. For plane wall:  $\theta_{(x,c)} = \frac{T_{(x,r)} - T_a}{T_i - T_a} = A_1 e^{-\lambda_1^2 Fo} \cos \frac{\lambda_1 x}{L}$  for  $Fo > 0.2$
2. Long cylinder:  $\theta_{(r,c)} = \frac{T_{(r,r)} - T_a}{T_i - T_a} = A_1 e^{-\lambda_1^2 Fo} J_0 \left( \frac{\lambda_1 r}{R} \right)$  for  $Fo > 0.2$
3. Sphere:  $\theta_{(r,c)} = \frac{T_{(r,r)} - T_a}{T_i - T_a} = A_1 e^{-\lambda_1^2 Fo} \frac{\sin \frac{\lambda_1 r}{R}}{\frac{\lambda_1 r}{R}}$  for  $Fo > 0.2$

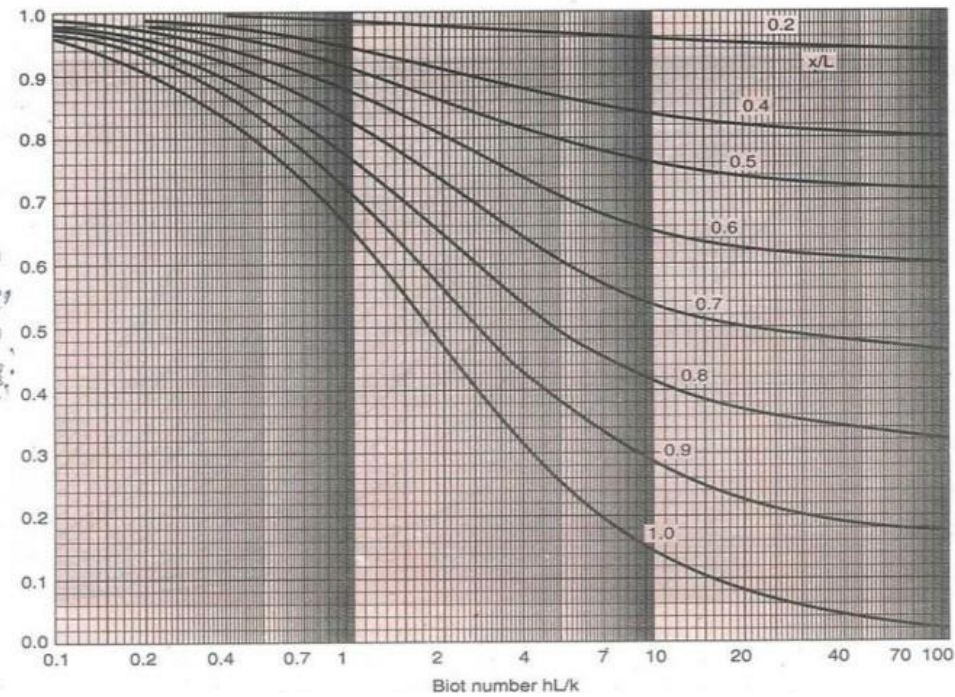
The one term approximation solutions were represented in graphical form by Heisler in 1947. They were supplemented by Grober in 1961, with graphs for the above heat transfer equations. These graphs are shown in figures below, for **plane wall**.



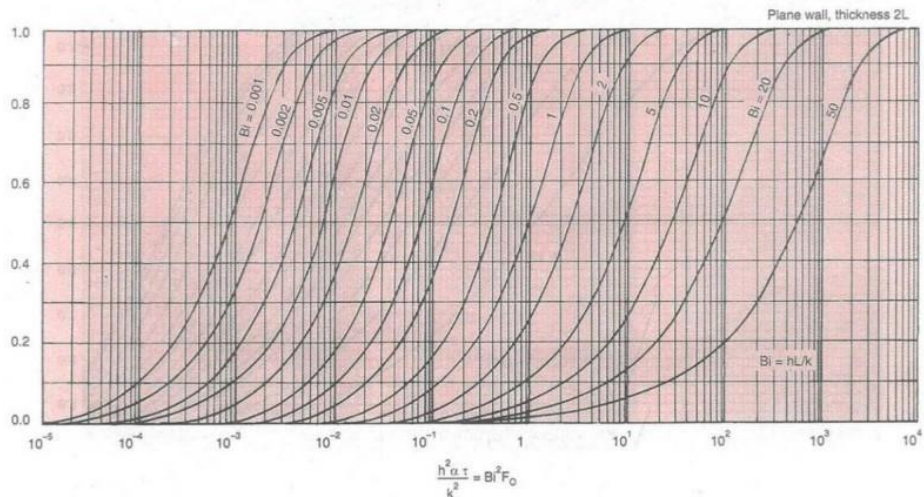
INFINITE PLATE—TEMPERATURE—TIME HISTORY AT MID PLANE



INFINITE PLATE—TEMPERATURE—TIME HISTORY AT ANY POSITION  $x$ ,



HEAT FLOW—INFINITE PLATE





## INTRODUCTION TO NUMERICAL ANALYSIS OF HEAT CONDUCTION

In module 1, we derived the general differential equation for heat conduction in cartesian, cylindrical and spherical coordinates. Subsequently, considering one-dimensional conduction, we solved these differential equations, with appropriate boundary conditions, for cases of simple geometries such as a plane wall, cylinder and sphere and obtained temperature distribution in those geometries; then, by applying Fourier's law, heat transfer rate was obtained. The analytical solutions obtained for temperature distribution are known as 'exact solutions' since temperature at any point in the solid is obtained by applying the equations derived. While getting an exact solution is always preferable, following points in connection with the analytical solutions must be noted:

(i) Analytical solutions are suitable for simple geometries such as a plane wall, cylinder or sphere, where the surface of the body and the coordinate surfaces coincide, i.e. surfaces of a plane wall are completely bounded by the coordinate surfaces of a cartesian coordinate system, surfaces of a cylinder and sphere are completely bounded by a cylindrical and spherical coordinate system respectively.

(ii) However, for irregular geometries, analytical solutions become difficult. For example, if there is a handle on a cylindrical cup, finding out the temperature distribution in the system becomes very difficult or impossible by analytical methods.

(iii) Further, even in simple geometries, if there is variation of thermal conductivity with temperature, or if the heat transfer coefficient varies over the surface, or if there is radiation heat transfer involved at the surfaces, severe non-linearities are introduced and analytical solutions become highly complicated or impossible.

(iv) Many times, analytical solutions, even if available for certain problems, are so complicated with the presence of infinite series, Bessel functions etc. that the user gets intimidated from using them. In such cases, popular alternative method is 'numerical solution'. Here, the differential equation is substituted by a set of algebraic equations and simultaneous solution of these algebraic equations gives the temperatures at selected, 'discrete points' in the system. So, the important difference to be noted is that while in an analytical solution, temperature is obtained at any point in the body, in a numerical solution temperature are obtained only at selected, discrete points or 'nodes'. By selecting these nodes close enough, sufficiently accurate results are obtained.

### **Advantages of numerical methods are:**

- (i) easy to apply, with the availability of high-speed computers
- (ii) desired accuracy can be obtained by controlling the number of nodes or 'mesh size'.
- (iii) variation in area, thermal conductivity or heat transfer coefficients, and complicated boundary conditions can be easily taken into account.
- (iv) mathematical model for a numerical solution is more likely to be a better representative of the actual system

(v) parametric study to observe the effect of variation of different parameters on the solution, or 'what-if' analysis, is easier with numerical methods in conjunction with high speed computers.

Generally used numerical techniques are 'finite difference', 'finite element', 'boundary element' and 'energy balance or control volume' methods. We will adopt energy balance method since it is intuitively easier to apply energy balance on control volumes and does not involve complicated mathematical formulations.

**Module 3: FORCED CONVECTION**

**Convective Heat Transfer:** - When a fluid flows over a body or inside a channel and if the temperatures of the fluid and the solid surface are different, heat transfer will take place between the solid surface and the fluid due to the macroscopic motion of the fluid relative to the surface. This mechanism of heat transfer is called as “*convective heat transfer*”. If the fluid motion is due to an external force (by using a pump or a compressor) the heat transfer is referred to as “*forced convection*”. If the fluid motion is due to a force generated in the fluid due to buoyancy effects resulting from density difference (density difference may be caused due to temperature difference in the fluid) then the mechanism of heat transfer is called as “*natural or free convection*”. For example, a hot plate suspended vertically in quiescent air causes a motion of air layer adjacent to the plate surface because the temperature gradient in the air gives rise to a density gradient which in turn sets up the air motion.

**Heat Transfer Coefficient:** - In engineering application, to simplify the heat transfer calculations between a hot surface say at temperature  $T_w$  and a cold fluid flowing over it at a bulk temperature  $T_\infty$  as shown in Figure 6.1 a term called “*heat transfer coefficient, h*” is defined by the equation

$$q = h(T_w - T_\infty)$$

where  $q$  is the heat flux (expressed in  $W / m^2$ ) from the surface to the flowing fluid. Alternatively if the surface temperature is lower than the flowing fluid then the heat transfer takes place from the hot fluid to the cold surface and the heat flux is given by

$$q = h(T_\infty - T_w)$$

The heat flux in this case takes place from the fluid to the cold surface. If in equations 6.1(a) and 6.1(b) the heat flux is expressed in  $W / m^2$ , then the units of heat transfer coefficient will be  $W \cdot m^{-2} \cdot K^{-1}$  or  $W \cdot m^{-2} \cdot ^\circ C^{-1}$ .

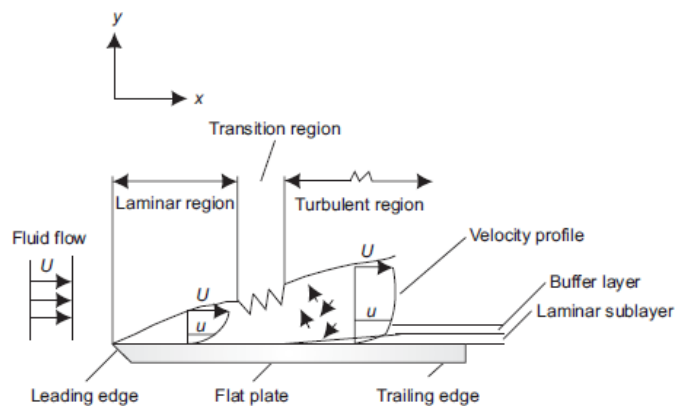
The heat transfer coefficient is found to vary with

- (i) The geometry of the body,
- (ii) The type of flow (laminar or turbulent),
- (iii) The transport properties of the fluid (density, viscosity, specific heat and thermal conductivity),
- (iv) The difference in temperature between the average temperature of the fluid and the surface with which the fluid is in contact,
- (v) The position along the surface of the body, and
- (vi) Whether the heat transfer is by forced convection or free convection.

For convection problems involving simple geometries like flow over a flat plate or flow inside a circular tube, the heat transfer coefficient can be determined analytically. But for flow over complex configurations, experimental / numerical approach is used to determine  $h$ . There is a wide difference in the range of values of  $h$  for various applications.

**Basic concepts for flow over a body:** - When a fluid flows over a body, the velocity and temperature distribution at the vicinity of the surface of the body strongly influence the heat transfer by convection. By introducing the concept of boundary layers (velocity boundary layer and thermal boundary layer) the analysis of convective heat transfer can be simplified.

**Velocity Boundary Layer:** - Consider the flow of a fluid over a flat plate as shown in figure. The fluid just before it approaches the leading edge of the plate has a velocity  $u_\infty$  which is parallel to the plate surface. As the fluid moves in  $x$ -direction along the plate, those fluid particles that makes contact with the plate surface will have the same velocity as that of the plate. Therefore, if the plate is stationary, then the fluid layer sticking to the plate surface will have zero velocity (This is called no-slip condition). But far away from the plate ( $y = \infty$ ) the fluid will have the velocity  $u_\infty$ . Therefore, starting from the plate surface ( $y = 0$ ) there will be retardation of the fluid in  $x$ -direction component of velocity  $u(x, y)$ . This retardation effect is reduced as we move away from the plate surface. At distances sufficiently long from the plate ( $y = \infty$ ) the retardation effect is completely reduced: i.e.  $u \rightarrow u_\infty$  as  $y \rightarrow \infty$ . This means that there is a region surrounding the plate surface where the fluid velocity changes from zero at the surface to the velocity  $u_\infty$  at the outer edge of the region. This region is called *the velocity boundary layer*. The variation of the  $x$ -component of velocity  $u(x, y)$  with respect to  $y$  at a particular location along the plate is shown in Figure 6.2. The distance measured normal to the surface from the plate surface to the point at which the fluid attains 99% of  $u_\infty$  is called "*velocity boundary layer thickness*" and denoted by  $\delta(x)$



- Thus for flow over a flat plate, the flow field can be divided into two distinct regions, namely,
- (i) *the boundary layer region* in which the axial component of velocity  $u(x, y)$  varies rapidly with  $y$  with the result the velocity gradient ( $\partial u / \partial y$ ) and hence the shear stress are very large and
  - (ii) *the potential flow region* which is outside the boundary layer region, where the velocity gradients and shear stresses are negligible.

The flow in the boundary layer, starting from the leading edge of the plate will be initially laminar in which the fluid particles move along a stream line in an orderly manner. In the laminar region the retardation effect is due to the viscosity of the fluid and therefore the shear stress can be evaluated using Newton's law of viscosity. The laminar flow continues along the plate until a critical distance ' $x_{cr}$ ' is reached. After this the small disturbances in the flow begin to grow and fluid fluctuations begin to develop. This characterizes the end of the laminar flow region and the beginning of transition from laminar to *turbulent boundary layer*. A dimensionless parameter called *Reynolds number* is used to characterize the flow as laminar or turbulent. For flow over a flat plate the Reynolds number is defined as

$$Re_x = \frac{u_\infty x}{\nu}$$

where  $u_\infty$  = free-stream velocity of the fluid,  $x$  = distance from the leading edge of the plate and  $\nu$  = kinematic viscosity of the fluid.

For flow over a flat plate it has been found that the transition from laminar flow to turbulent flow takes place when the Reynolds number is  $\approx 5 \times 10^5$ . This number is called as the critical Reynolds number  $Re_{cr}$  for flow over a flat plate. Therefore

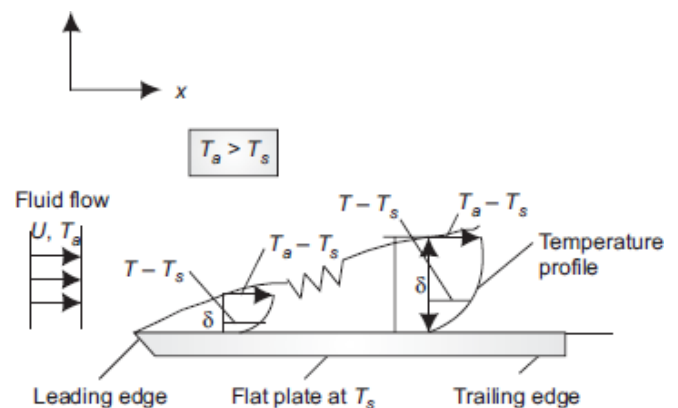
$$Re_{cr} = \frac{u_\infty X_{cr}}{\nu} = 5 \times 10^5$$

The critical Reynolds number is strongly dependent on the surface roughness and the turbulence level of the free stream fluid. For example, with very large disturbances in the free stream, the transition from laminar flow to turbulent flow may begin at  $Re_x$  as low as  $1 \times 10^5$  and for flows which are free from disturbances and if the plate surface is smooth transition may not take place until a Reynolds number of  $1 \times 10^6$  is reached. But it has been found that for flow over a flat plate the boundary layer is always turbulent for  $Re_x \geq 4 \times 10^6$ . In the turbulent boundary layer next to the wall there is a very thin layer called "*the viscous sub-layer*", where the flow retains its viscous flow character. Next to the viscous sub-layer is a region called "*buffer layer*" in which the effect of fluid viscosity is of the same order of magnitude as that of turbulence and the mean velocity rapidly increases with the distance from the plate surface. Next to the buffer layer is "*the turbulent layer*" in which there is large scale turbulence and the velocity changes relatively little with distance.

**Thermal boundary layer:** - Similar to the velocity boundary layer one can visualize the development of a thermal boundary layer when a fluid flows over a flat plate with the temperature of the plate being different from that of the free stream fluid. Consider that a fluid at a uniform temperature  $T_\infty$  flows over a flat plate which is maintained at a uniform temperature  $T_w$ . Let  $T(x, y)$  is the temperature of the fluid at any location in the flow field. Let the dimensionless temperature of the fluid  $\theta(x, y)$  be defined as

$$\theta(x, y) = \frac{T(x, y) - T_w}{T_\infty - T_w}$$

The fluid layer sticking to the plate surface will have the same temperature as the plate surface [ $T(x, y)_{y=0} = T_w$ ] and therefore  $\theta(x, y) = 0$  at  $y = 0$ . Far away from the plate the fluid temperature is  $T_\infty$  and hence  $\theta(x, y) \rightarrow 1$  as  $y \rightarrow \infty$ . Therefore at each location  $x$  along the plate one can visualize a location  $y = \delta_t(x)$  in the flow field at which  $\theta(x, y) = 0.99$ .  $\delta_t(x)$  is called "*the thermal boundary layer thickness*" as shown in Figure 6.3. The locus of such points at which  $\theta(x, y) = 0.99$  is called the edge of the thermal boundary layer. The relative thickness of the thermal boundary layer  $\delta_t(x)$  and the velocity boundary layer  $\delta(x)$  depends on a dimensionless number called "*Prandtl number*" of the fluid. It is denoted by  $Pr$  and is defined as



$$Pr = \frac{\mu C_p}{k} = \frac{(\mu/\rho)}{(k/\rho C_p)} = \frac{\nu}{\alpha}$$

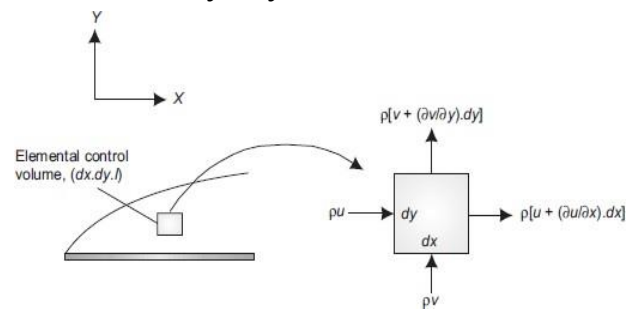
Where  $\mu$  is the absolute viscosity of the fluid,  $C_p$  is the specific heat at constant pressure,  $k$  is the thermal conductivity,  $\nu$  is the kinematic viscosity and  $\alpha$  is the thermal diffusivity of the fluid. The Prandtl number for fluids ranges from 0.01 for liquid metals to more than 100,000 for heavy oils. For fluids with  $Pr = 1$  such as gases  $\delta_t(x) = \delta(x)$ , for fluids with  $Pr \ll 1$ , such as liquid metals,  $\delta_t(x) \gg \delta(x)$  and for fluids with  $Pr \gg 1$ , like oils  $\delta_t(x) \ll \delta(x)$ .

**Differential Equations for the Boundary Layer:** In convection studies, since there is a fluid flow, we are interested in the shear stress and the friction coefficient; to determine these we need the velocity gradient at the surface. Similarly, to determine the convection coefficient, we need the temperature gradient at the surface. To determine the velocity gradient at the surface, we apply the equation of conservation of momentum (in conjunction with the equation of conservation of mass) to a differential volume element in the boundary layer. And, to determine the temperature gradient at the surface, we apply the equation of conservation of energy to a differential volume element in the boundary layer.

**Conservation of Mass, The Continuity Equation for The Boundary Layer:** Consider a differential control volume, of section  $(dx, dy)$  and unit depth, within the boundary layer, as shown in figure.

**Assumptions:**

- (i) Flow is steady, incompressible
- (ii) Constant fluid properties
- (iii) Pressure variation is only in the X-direction
- (iv) Shear in Y-direction is negligible
- (v) Continuity in space and time



Let  $u$  and  $v$  be the velocity components in the  $X$  and  $Y$ -directions. Then, remembering that the mass flow rate is given by (density x velocity x area) and that the depth is unity in the  $Z$ -direction, we can write:

Mass flow into the control volume in  $X$ -direction =  $\rho \cdot u \cdot (dy \cdot 1)$

Mass flow out of the control volume in  $X$ -direction =  $\rho \cdot [u + (\partial u/\partial x) \cdot dx] \cdot (dy \cdot 1)$

Therefore, net mass flow into the element in the  $X$ -direction =  $-\rho \cdot (\partial u/\partial x) \cdot dx \cdot dy$

Similarly, net mass flow into the control volume in the  $Y$ -direction is =  $-\rho \cdot (\partial v/\partial y) \cdot dy \cdot dx$

Since the net mass flow into control volume, in steady state, must be equal to zero, we write:

$$-\rho \cdot \{(\partial u/\partial x) + (\partial v/\partial y)\} \cdot dx \cdot dy = 0$$

i.e. for a two-dimensional flow in the boundary layer, equation of conservation of mass is given by:

$$(\partial u/\partial x) + (\partial v/\partial y) = 0$$

The above equation is known as '**continuity equation**' for two-dimensional, steady flow of an incompressible fluid.

### Conservation of Energy Equation for The Boundary Layer:

#### Assumptions:

- (i) steady, incompressible flow
- (ii) conduction is only in the Y-direction
- (iii) temperature change in the X-direction is small i.e. negligible conduction in flow direction
- (iv) specific heat ( $C_p$ ) of the fluid is constant
- (v) negligible viscous heating
- (vi) negligible body forces

Figure shows the rate at which energy is conducted and convected into and out of the differential control volume.

Note that in addition to the conductive terms, there are four convective terms.

Let us write the different energy terms and apply the energy balance which states that net rate of conduction and convection should be equal to zero:

*Convective terms:*

For the X-direction:

Energy into the control volume =  $\rho.C_p.u.T.dy$

Energy out of the control volume =  $\rho.C_p.\{u + (\partial u/\partial x).dx\}.\{T + (\partial T/\partial x) dx\}.dy$

Therefore, neglecting the product of differentials, net energy convected into the control volume in the x-direction is given by:  $-\rho.C_p.\{u.(\partial T/\partial x) + T.(\partial u/\partial x)\}.dx.dy$

Similarly, net energy convected into the control volume in the Y-direction is given by:  $-\rho.C_p.\{v.(\partial T/\partial y) + T.(\partial v/\partial y)\}.dx.dy$

*Conductive terms:*

Conduction in Y-direction.

Net conduction into the control volume in the Y-direction is given by:

$$-k.dx.(\partial T/\partial y) - [-k.dx.\{(\partial T/\partial y) + (\partial^2 T/\partial y^2).dy\}] = k.(\partial^2 T/\partial y^2).dx.dy$$

Similarly, for completeness, net conduction into the control volume in the X-direction is given by:

$$k.(\partial^2 T/\partial x^2).dx.dy$$

When the viscous work is neglected, making an energy balance and using continuity equation energy balance equation becomes,

$$u.(\partial T/\partial x) + v.(\partial T/\partial y) = (k/r.C_p).(\partial^2 T/\partial y^2)$$

This is the energy equation for a two-dimensional, steady incompressible flow, when the viscous dissipation is neglected, i.e. for very low velocities of flow.

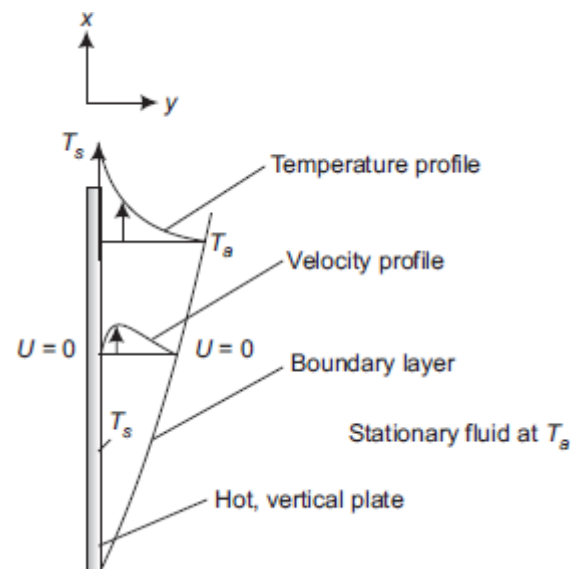
**NOTE:** For **Various empirical solutions, forced convection flow over cylinders and spheres, Internal flows – laminar and turbulent flow solutions** please refer to the Heat Transfer Data Book.

## FREE CONVECTION

Here, fluid movement is caused because of density differences in the fluid due to temperature differences, under the influence of gravity. Density differences cause a 'buoyancy force' which in turn, causes the fluid circulation by 'convection currents'. Buoyancy force is the upward force exerted by a fluid on a completely or partially immersed body and is equal to the weight of the fluid displaced by the body. Natural convection is one of the important modes of heat transfer used in practice since there are no moving parts and as a result, there is an increased reliability. Natural convection heat transfer is extensively used in the following areas of engineering:

- (i) cooling of transformers, transmission lines and rectifiers
- (ii) heating of houses by steam or electrical radiators
- (iii) heat loss from steam pipe lines in power plants and heat gain in refrigerant pipe lines in air-conditioning applications
- (iv) cooling of reactor core in nuclear power plants
- (v) cooling of electronic devices (chips, transistors, etc.) by finned heat sinks.

**Physical Mechanism of Natural Convection:** Consider the familiar example of a heated, vertical plate kept hanging in quiescent air. Let the temperature of the heated surface be  $T_s$  and that of the surrounding air,  $T_a$ . A layer of air in the immediate vicinity of the plate will get heated by conduction; density of this heated air layer decreases (since the total pressure of surroundings is constant and  $p = \rho \times R_{air} T$  for an ideal gas). As a result, the heated layer rises up and the cold air from the surroundings moves in to take its place. This layer, in turn, gets heated up, moves up and is again replaced by cooler air etc. Thus, convection currents are set up causing the heat to be carried away from the hot surface. This situation is shown in figure.

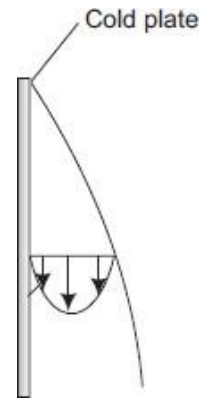


During the temperature induced flow, a boundary layer is set up along the length of the plate as shown. With the x-axis taken along the vertical length of plate, and the y-axis perpendicular to it, the velocity and temperature profiles are shown in the figure. As far as the velocity profile is concerned, at the plate surface, the fluid velocity is zero due to 'no slip' condition; then, the velocity increases to a maximum value and then, drops to zero at the outer edge of the boundary layer since the surrounding air is assumed to be quiescent. Note the difference in this velocity profile as compared to that in the case of forced convection. The boundary layer is laminar for some distance along the length, and then depending on the fluid properties and the driving temperature difference between the wall and the ambient, the boundary layer becomes turbulent.



**Empirical Relations for Natural Convection Over Surfaces and Enclosures:**

**Vertical Plate at Constant Temperature  $T_s$ :** Vertical plate is an important geometry since heat transfer from the walls of a furnace can be calculated by the relations applicable to a vertical plate. McAdams has suggested the following relations for fluids whose Prandtl number is close to unity, i.e. for air and other gases, generally:



$$Nu = 0.59 \times Ra^{\frac{1}{4}} \dots\dots\dots 10^4 < Ra < 10^9$$

and  $Nu = 0.13 \times Ra^{\frac{1}{3}} \dots\dots\dots 10^9 < Ra < 10^{12}$

first equation is for laminar, boundary layer type, natural convection flow, while second equation is for turbulent, boundary layer type, natural convection flows. Fluid properties are evaluated at film temperature  $T_f$ , already defined.

Churchill and Chu present following relations for the entire range of Ra and also valid for all Prandtl numbers from 0 to  $\infty$ .

For  $0 < Ra < 10^9, 0 < Pr < \infty$ :

$$Nu = 0.68 + \frac{0.670 \cdot Ra^{\frac{1}{4}}}{\left[1 + \left(\frac{0.492}{Pr}\right)^{\frac{9}{16}}\right]^{\frac{4}{9}}} \dots\dots\dots (0 < Ra < 10^9)$$

For  $Ra > 10^9, 0.6 < Pr < \infty$ :

$$Nu = \frac{0.15 \cdot Ra^{\frac{1}{3}}}{\left[1 + \left(\frac{0.492}{Pr}\right)^{\frac{9}{16}}\right]^{\frac{16}{27}}} \dots\dots\dots (Ra > 10^9)$$

For  $Ra > 10^9, 0 < Pr < 0.6$ :

$$Nu = \left[0.825 + \frac{0.387 \cdot Ra^{\frac{1}{4}}}{\left[1 + \left(\frac{0.492}{Pr}\right)^{\frac{9}{16}}\right]^{\frac{8}{27}}}\right]^2 \dots\dots\dots (Ra > 10^9)$$

**Vertical Cylinders at Constant Temperature  $T_s$ :** A vertical cylinder can be treated as a vertical plate and the relations given above can be applied if the following criterion is satisfied:

$$\frac{D}{L} \geq \frac{34}{Ra^{\frac{1}{4}}}$$

Height  $L$  of the cylinder is the characteristic dimension

**Vertical Plate with Constant Heat Flux:** Equations of Churchill and Chu, 10.25 and 10.26 are valid, with the following modifications: (a) temperature of the constant flux plate is considered at a point mid-way between top and bottom (b) constant 0.492 should be changed to 0.437. Alternative relations are given below for vertical and inclined plates for natural convection in water and air.

Here, a modified Grashoff number,  $Gr'$  is defined:  $Gr' = Gr \cdot Nu_x = \frac{g \cdot \beta \cdot q_s \cdot x^4}{k \cdot \nu^2}$

where  $q_s$  is the wall heat flux in  $W/m^2$ . Then the following two relations are recommended for local heat transfer coefficients in laminar and turbulent ranges respectively:

$$Nu_x = 0.60 \times (Gr' \times Pr)^{0.2} \quad (10^5 < Gr' < 10^{11})$$

and, 
$$Nu_x = 0.17 \times (Gr' \times Pr)^{0.25} \quad (Gr' > 10^{11})$$

And the average heat transfer coefficient in the laminar region is obtained by integration over the entire height  $L$  of the plate as:

$$h = \frac{5}{4} h_L \quad (\text{for laminar})$$

and, for turbulent region,  $h_x$  is independent of  $x$ :  $h = h_L$  (for turbulent)

**Horizontal Plate at Constant Temperature  $T_s$ :** Here, the characteristic length to be used in expressions for  $Nu$  and  $Gr$  is:

$$L_c = A/P$$

where,  $A$  is the surface area and  $P$  is the perimeter.

Property values are evaluated at film temperature,  $T_f$ .

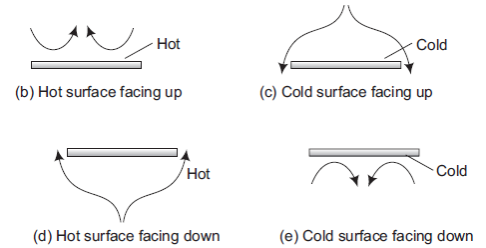
(a) Upper surface of a hot plate (or, lower surface of a cold plate):

$$Nu = 0.54 Ra^{\frac{1}{4}} \quad (10^4 < Ra < 10^7)$$

and, 
$$Nu = 0.15 Ra^{\frac{1}{3}} \quad (10^7 < Ra < 10^{11})$$

(b) Lower surface of a hot plate (or upper surface of a cold plate):

$$Nu = 0.27 \times Ra^{\frac{1}{4}} \quad (10^5 < Ra < 10^{11})$$



**Horizontal Plate With Constant Heat Flux:** Here, the characteristic length to be used in expressions for  $Nu$  and  $Gr$  is:

$$L_c = A/P$$

where,  $A$  is the surface area and  $P$  is the perimeter.

For a circle,  $L_c = 0.9D$ , and for rectangle,  $L_c = (L + W)/2$ .

All property values, except  $\beta$ , are evaluated at a temperature,  $T_e$ , defined by:

$$T_e = T_s - 0.25 (T_s - T_a)$$

and,  $\beta$  is evaluated at  $T_a$ .

$T_s$  is estimated from the basic relation:

$$h_{avaa} (T_s - T_a) = q_s$$

(a) Upper surface of a hot plate (or lower surface of a cold plate):

$$Nu = 0.13 \times Ra^{\frac{1}{3}} \quad (Ra < 2 \times 10^8)$$

and, 
$$Nu = 0.16 \times Ra^{\frac{1}{3}} \quad (2 \times 10^8 < Ra < 10^{11})$$

(b) For heated surface facing downward:

$$Nu = 0.58 \times Ra^{0.2} \quad (10^6 < Ra < 10^{11})$$

As in the case of vertical plates with constant heat flux, in this case also, iteration will be required while solving problems.

**Horizontal Cylinder at Constant Temperature:**

Here,  $D$ , diameter of the cylinder is the characteristic dimension.

For heat transfer from (or to) a horizontal cylinder, Morgan recommends following correlation for fluids with  $(0.69 < Pr < 7)$ :

$$Nu = C \times Ra^n$$

where  $C$  and  $n$  are obtained from the following Table:

Ra	C	n
$10^{-10}$ – $10^{-2}$	0.675	0.058
$10^{-2}$ – $10^2$	1.02	0.148
$10^2$ – $10^4$	0.85	0.188
$10^4$ – $10^7$	0.48	0.25
$10^7$ – $10^{12}$	0.125	0.333

Also, the following correlation of Churchill and Chu may be used for the complete range of Prandtl numbers:  $(0 \leq Pr \leq \infty)$  and for a wider range of Rayleigh numbers:

$$Nu = \left[ 0.60 + 0.387 \cdot \frac{Ra}{\left[ 1 + \left( \frac{0.559}{Pr} \right)^{\frac{9}{16}} \right]^{\frac{1}{4}}} \right]^{\frac{1}{4}} \quad (10^{-5} < Ra < 10^{12})$$

And, only for the laminar range:

$$Nu = 0.36 + \frac{0.518 \cdot Ra^{\frac{1}{4}}}{\left[ 1 + \left( \frac{0.559}{Pr} \right)^{\frac{9}{16}} \right]^{\frac{1}{4}}} \quad (10^{-6} < Ra < 10^9)$$

Properties in the above equations are evaluated at film temperature,  $D$  is the characteristic dimension. Churchill and Chu recommend that above two eqns. may be used for constant flux conditions too, with the temperature  $T_s$  being half way up the cylinder at the 90 deg. angle from bottom.

**For thin wires:** ( $D = 0.2$  mm to 1 mm): Rayleigh number is usually very small and a film type of flow pattern is observed. Following correlation is used:

$$Nu_D = 1.18 \times (Ra_D^{\frac{1}{8}}) \quad (Ra < 500)$$

Heat transfer from horizontal cylinders to liquid metals may be calculated from:

$$Nu_D = 0.53 \times (Gr_D \times Pr^2)^{\frac{1}{4}}$$

## Module 4: THERMAL RADIATION

When a body is placed in an enclosure whose walls are at temperatures below that of the body, the temperature of the body will decrease even if the enclosure is evacuated. This process by which heat is transferred from a body by virtue of its temperature, without the aid of any intervening medium is called "Thermal Radiation". The actual mechanism of radiation is not yet completely understood. There are at present two theories by means of which radiation propagation is explained. According to Maxwell's electromagnetic theory, Radiation is treated as electromagnetic waves, while Max Planck's theory treats radiation as "Photons" or "Quanta of energy". Neither theory completely describes all observed phenomena. It is however known that radiation travels with the speed of light,  $c$  ( $c = 3 \times 10^8$  m/s) in a vacuum. This speed is equal to the product of the frequency of the radiation and the wavelength of this radiation,

or

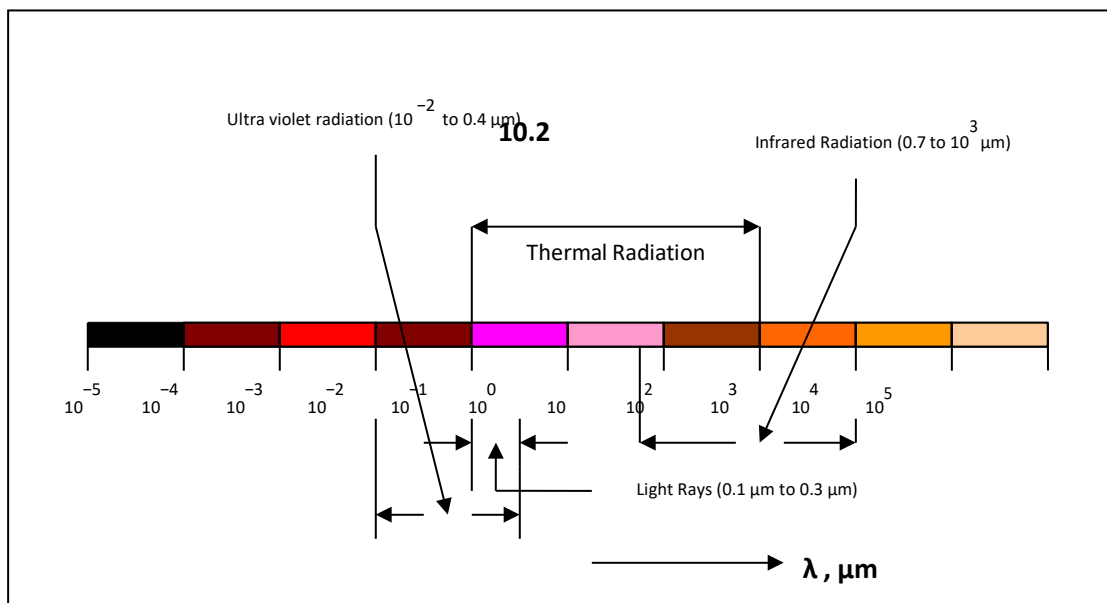
$$c = \lambda \nu$$

where  $\lambda$  = wavelength of radiation (m) and  $\nu$  = frequency (1/s).

Usually, it is more convenient to specify wavelength in micro meter, which is equal to  $10^{-6}$  m.

From the viewpoint of electromagnetic wave theory, the waves travel at the speed of light, while from the quantum theory point of view, energy is transported by photons which travel at the speed of light. Although all the photons have the same velocity, there is always a distribution of energy among them. The energy associated with a photon,  $e_p = h\nu$  where  $h$  is the Planck's constant equal to  $6.6256 \times 10^{-34}$  Js. The entire energy spectrum can also be described in terms of the wavelength of radiation.

Radiation phenomena are usually classified by their characteristic wavelength,  $\lambda$ . At temperatures encountered in most engineering applications, the bulk of the thermal energy emitted by a body lies in the wavelengths between  $\lambda = 0.1$  and  $100 \mu\text{m}$ . For this reason, the portion of the wavelength spectrum between  $\lambda = 0.1$  and  $100 \mu\text{m}$  is generally referred to as "Thermal Radiation". The wavelength spectrum in the range  $\lambda = 0.4$  and  $0.7 \mu\text{m}$  is visible to the naked eye, and this is called 'light rays'. The wavelength spectrum of radiation is illustrated in figure below.



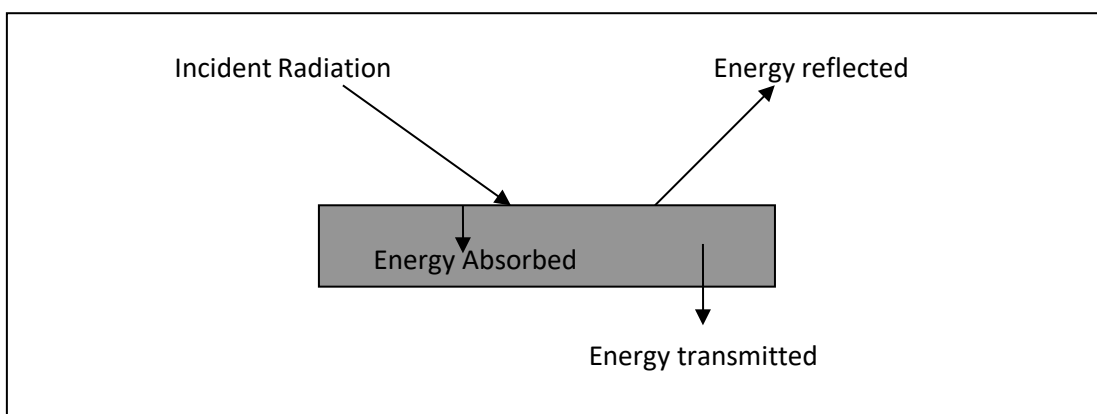
In the study of radiation transfer, a distinction should be made between bodies which are “semi-transparent” to radiation and those which are “opaque”. If the material is semi-transparent to radiation, such as glass, salt crystals, and gases at elevated temperatures, then the radiation leaving the body from its outer surfaces results from emissions at all depths within the material. The emission of radiation for such cases is a “Bulk” or a “Volumetric Phenomenon”. If the material is opaque to thermal radiation, such as metals, wood, rock etc. then the radiation emitted by the interior regions of the body cannot reach the surface. In such cases, the radiation emitted by the body originates from the material at the immediate vicinity of the surface (i.e. within about  $1\mu\text{m}$ ) and the emission is regarded as a “Surface Phenomenon”. It should also be noted that a material may behave as a semi-transparent medium for certain temperature ranges, and as opaque for other temperatures. Glass is a typical example for such behaviour. It is semi-transparent to thermal radiation at elevated temperatures and opaque at intermediate and low temperatures.

### Definitions of Terms used in Thermal Radiation:

- **Monochromatic Emissive Power ( $E_\lambda$ ):** The monochromatic emissive power of a surface at any temperature  $T$  and wavelength  $\lambda$  is defined as the quantity which when multiplied by  $d\lambda$  gives the radiant flux in the wavelength range -  $\lambda$  to  $\lambda+d\lambda$ .
- **Emissive Power (E):** The emissive power of a surface is the energy emitted by a surface at a given temperature per unit time per unit area for the entire wavelength range, from  $\lambda = 0$  to  $\lambda = \infty$ .

$$E = \int_0^{\infty} E_\lambda d\lambda$$

- **Absorptivity, Reflectivity and Transmissibility of a body:**



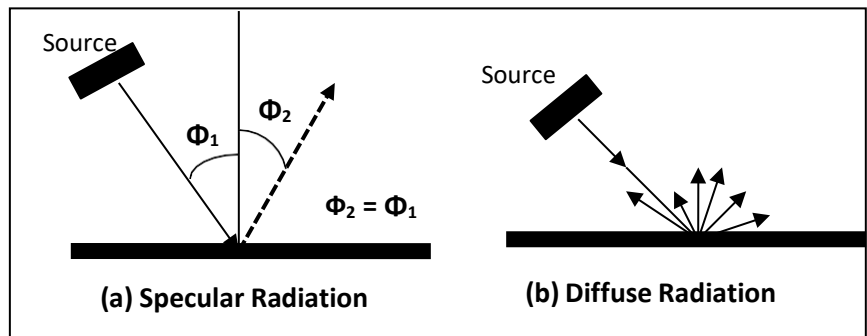
When a radiant energy strikes a material surface, part of the radiation is reflected, part is absorbed, and part is transmitted, as shown in above figure. Reflectivity ( $\rho$ ) is defined as the fraction of energy which is reflected, Absorptivity ( $\alpha$ ) as the fraction absorbed, and Transmissivity ( $\tau$ ) as the fraction transmitted. Thus,  $\rho + \alpha + \tau = 1$ .

Most solid bodies do not transmit thermal radiation, so that for many applied problems, the transmissivity may be taken as zero. Then

$$\rho + \alpha = 1$$

➤ **Specular Radiation and Diffuse Radiation:**

When radiation strikes a surface, two types of reflection phenomena may be observed. If the angle of incidence is equal to the angle of reflection, the radiation is called Specular. On the other hand, when an incident



beam is distributed uniformly in all directions after reflection, the radiation is called Diffuse Radiation. The two types of radiation are depicted in figure. Ordinarily, no real surface is either specular or diffuse. An ordinary mirror is specular for visible light, but would not necessarily be specular over the entire wavelength range. A rough surface exhibits diffuse behaviour better than a highly polished surface. Similarly, a highly polished surface is more specular than a rough surface.

➤ **Black Body:** A body which absorbs all incident radiation falling on it is called a blackbody. For a blackbody,  $\alpha = 1, \rho = \tau = 0$ . For a given temperature and wavelength, no other body at the same temperature and wavelength, can emit more radiation than a blackbody. Blackbody radiation at any temperature T is the maximum possible emission at that temperature. A blackbody or ideal radiator is a theoretical concept which sets an upper limit to the emission of radiation. It is a standard with which the radiation characteristics of other media are compared.

➤ **Emissivity of a Surface ( $\epsilon$ ):** The emissivity of a surface is the ratio of the emissive power of the surface to the emissive power of a black surface at the same temperature. It is denoted by the symbol  $\epsilon$ .

i.e. 
$$\epsilon = [E/E_b]_T$$

➤ **Monochromatic Emissivity of a Surface ( $\epsilon_\lambda$ ):** The monochromatic emissivity of a surface is the ratio of the monochromatic emissive power of the surface to the monochromatic emissive power of a black surface at the same temperature and same wavelength.

$$\epsilon_\lambda = [E_\lambda / E_{b\lambda}]_{\lambda, T}$$

➤ **Gray Body:** A gray body is a body having the same value of monochromatic emissivity at all wavelengths. i.e.

$$\epsilon = \epsilon_\lambda \text{ for a gray body.}$$

➤ **Radiosity of a Surface (J):** This is defined as the total energy leaving a surface per unit time per unit area of the surface. This definition includes the energy reflected by the surface due to some radiation falling on it.

➤ **Irradiation of a surface(G):** This is defined as the radiant energy falling on a surface per unit time, per unit area of the surface. Therefore, if E is the emissive power, J is the radiosity, ε is the irradiation and ρ the reflectivity of a surface, then,

$$J = E + \rho G$$

For an opaque surface,  $\rho + \alpha = 1$  or  $\rho = (1 - \alpha)$

$$J = E + (1-\alpha)G$$

**Laws of Radiation:**

**STEFAN – BOLTZMANN LAW:**

This law states that the emissive power of a blackbody is directly proportional to the fourth power of the absolute temperature of the body.

i.e.,  $E_b \propto T^4$

or  $E_b = \sigma T^4$

where σ is called the Stefan – Boltzmann constant.

In SI units  $\sigma = 5.669 \times 10^{-8} \text{ W}/(\text{m}^2\text{-K}^4)$ .

**PLANCK’S LAW:**

This law states that the monochromatic power of a blackbody is given by

$$E_{b\lambda} = \frac{C_1}{\lambda^5 [e^{(C_2/\lambda T)} - 1]}$$

where C<sub>1</sub> and C<sub>2</sub> are constants whose values are found from experimental data;

C<sub>1</sub> = 3.7415 × 10<sup>-16</sup> Wm<sup>2</sup> and C<sub>2</sub> = 1.4388 × 10<sup>-2</sup> m-K.

λ is the wavelength and T is the absolute temperature in K.

**WEIN’S DISPLACEMENT LAW:**

It can be seen from Eq. 10.6 that at a given temperature, E<sub>bλ</sub> depends only on λ. Therefore the value of λ which gives maximum value of E<sub>bλ</sub> can be obtained by differentiating it w.r.t λ and equating it to zero.

Let  $C_2/\lambda T = y$ . Then Eq. (10.6) reduces to

$$E_{b\lambda} = \frac{C_1}{[C_2 / (yT)]^5 [e^y - 1]}$$

Then 
$$\frac{dE_{b\lambda}}{dy} = \frac{C_1 d / dy \{ [C_2 / (yT)]^5 [e^y - 1] \}}{\{ [C_2 / (yT)]^5 [e^y - 1] \}^2}$$

or 
$$d / dy \{ [C_2 / (yT)]^5 (e^y - 1) \} = 0$$
  
 or 
$$e^y(5 - y) = 5$$

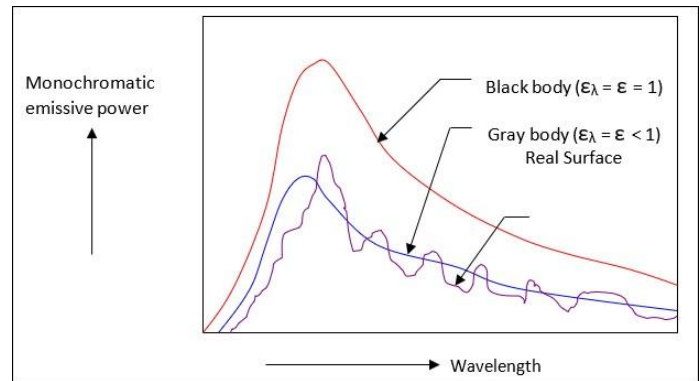
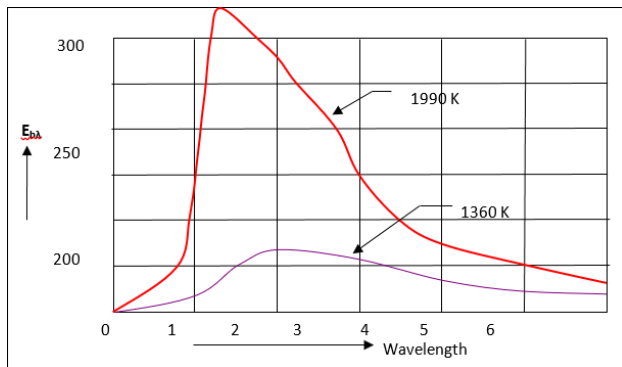
Therefore, if  $\lambda_m$  denotes the value of  $\lambda$  which gives maximum  $E_{b\lambda}$ , then

$$C_2/\lambda_m T = 4.965$$

or  $\lambda_m T = C_2/4.965 = 1.4388 \times 10^{-2} / 4.965$

$$\lambda_m T = 0.002898 \text{ m-K}$$

This equation is called the Wein's displacement law. From this equation it can be seen that the wavelength at which the monochromatic emissive power decreases with increasing temperature. This is also illustrated in the following figures gives a comparison of monochromatic emissive powers for different surfaces at a particular temperature for different wavelengths.



### KIRCHOFF'S LAW:

This law states that the emissivity of a surface is equal to its absorptivity when the surface is in thermal equilibrium with the surroundings.

**Proof:** Consider a perfect black enclosure i.e. the one which absorbs all the incident radiation falling on it (see figure below). Now let the radiant flux from this enclosure per unit area arriving at some area be  $q_i$  W/m<sup>2</sup>.

Now suppose that a body is placed inside the enclosure and allowed to come to thermal equilibrium with it. At equilibrium, the energy absorbed by the body must be equal to the energy emitted; otherwise there would be an energy flow into or out of the body, which would raise or lower its temperature. At thermal equilibrium we may write

$$EA = q_i A \alpha \quad (a)$$

If we now replace the body in the enclosure with a black body of the same size and shape and allow it to come to thermal equilibrium with the enclosure,

$$E_b A = q_i A \quad (b)$$

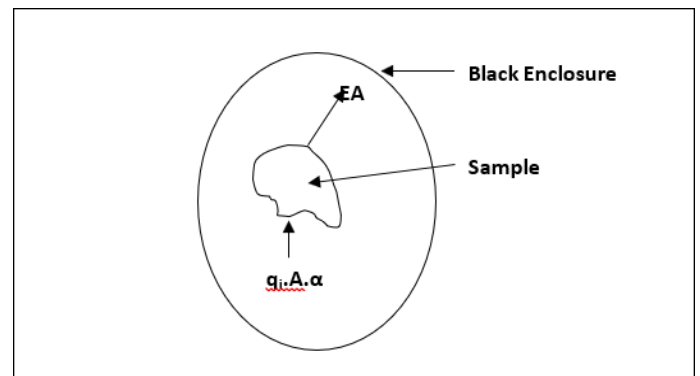
Since  $\alpha = 1$  for a blackbody.

If Eq. a is divided by Eq. b we get

$$E/E_b = \alpha$$

But by definition  $E/E_b = \epsilon$ , the emissivity of the body, so that

$$\epsilon = \alpha$$





the above equation is called Kirchoff's law and is valid only when the body is in thermal equilibrium with the surroundings. However, while analysing radiation problems in practice we assume that Kirchoff's law holds good even if the body is not in thermal equilibrium with the surroundings, as the error involved is not very significant.

**Radiation Heat Exchange Between Infinite Parallel Surfaces in The Presence of Non-Participating Medium:**

**Assumptions:**

- (i) The medium does not participate in radiation heat exchange between the two surfaces.
- (ii) The surfaces are flat and are at specified uniform temperatures.

**Radiation Exchange Between Two Parallel Black Surfaces:**

Since both surfaces are parallel, flat and infinite, radiosity of surface 1 = irradiation of surface 2 and vice versa. i.e.  $J_1 = G_2$  and  $J_2 = G_1$ .

Since both the surfaces are black,

$$J_1 = E_{b1} = \sigma T_1^4 \text{ and } J_2 = E_{b2} = \sigma T_2^4$$

$$\text{Net radiation leaving } A_1 = Q_{r1} = A_1(J_1 - G_1)$$

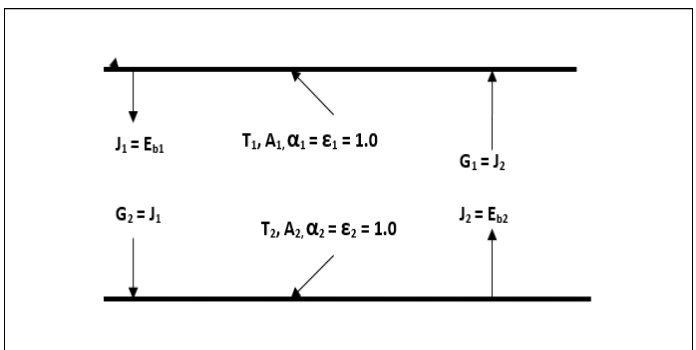
All this energy will reach  $A_2$ .

Net radiation leaving  $A_1$  and reaching  $A_2$  is given by

$$Q_{1-2} = Q_{r1} = A_1(J_1 - G_1) = A_1[J_1 - J_2]$$

or  $Q_{1-2} = A_1[E_{b1} - E_{b2}]$

or  $Q_{1-2} = \sigma A_1[T_1^4 - T_2^4]$



**Radiation Heat Exchange Between Two Parallel Infinite Gray surfaces:**

Since the net radiation leaving  $A_1$  will reach  $A_2$ ,

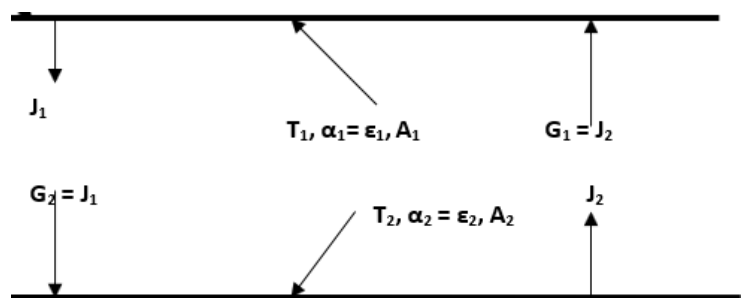
$$Q_{1-2} = Q_{r1} = A_1[J_1 - G_1]$$

$$J_1 = E_1 + (1 - \alpha_1)G_1$$

$$J_2 = E_2 + (1 - \alpha_2)G_2$$

$$J_1 = G_2$$

$$J_2 = G_1$$



The above equation can be written as

$$J_1 - (1 - \alpha_1)G_1 = E_1$$

Equation can be rewritten as

$$-(1 - \alpha_2)J_1 + G_1 = E_2$$

Solving for  $J_1$  and  $G_1$  from the above equations we get

$$J_1 = \frac{E_1 + (1 - \alpha_1) E_2}{1 - (1 - \alpha_1) (1 - \alpha_2)}$$

$$\epsilon_1 E_{b1} + (1 - \alpha_1) \epsilon_2 E_{b2}$$

$$\text{or } J_1 = \frac{\epsilon_1 E_{b1} + (1 - \alpha_1) \epsilon_2 E_{b2}}{1 - (1 - \alpha_1)(1 - \alpha_2)}$$

$$\epsilon_2 E_{b2} + (1 - \alpha_2) \epsilon_1 E_{b1}$$

$$\text{and } G_1 = \frac{\epsilon_2 E_{b2} + (1 - \alpha_2) \epsilon_1 E_{b1}}{1 - (1 - \alpha_1)(1 - \alpha_2)}$$

Substituting these expressions for  $J_1$  and  $G_1$  in above equation we get

$$A_1$$

$$Q_{1-2} = \frac{A_1 [\epsilon_1 E_{b1} + (1 - \alpha_1) \epsilon_2 E_{b2} - \epsilon_2 E_{b2} - (1 - \alpha_2) \epsilon_1 E_{b1}]}{[1 - (1 - \alpha_1)(1 - \alpha_2)]}$$

$$A_1 [\alpha_2 \epsilon_1 E_{b1} - \alpha_1 \epsilon_2 E_{b2}]$$

$$\text{or } Q_{1-2} = \frac{A_1 [\alpha_2 \epsilon_1 E_{b1} - \alpha_1 \epsilon_2 E_{b2}]}{[1 - (1 - \alpha_1)(1 - \alpha_2)]}$$

$$[1 - (1 - \alpha_1)(1 - \alpha_2)]$$

Substituting for  $E_{b1}$  and  $E_{b2}$  in terms of temperatures we get

$$\sigma A_1 [\alpha_2 \epsilon_1 T_1^4 - \alpha_1 \epsilon_2 T_2^4]$$

$$\text{or } Q_{1-2} = \frac{\sigma A_1 [\alpha_2 \epsilon_1 T_1^4 - \alpha_1 \epsilon_2 T_2^4]}{[1 - (1 - \alpha_1)(1 - \alpha_2)]}$$

$$[1 - (1 - \alpha_1)(1 - \alpha_2)]$$

If Kirchoff's law holds good then  $\alpha_1 = \epsilon_1$  and  $\alpha_2 = \epsilon_2$ .

$$\sigma A_1 [\epsilon_1 \epsilon_2 T_1^4 - \epsilon_1 \epsilon_2 T_2^4]$$

$$\text{Hence } Q_{1-2} = \frac{\sigma A_1 [\epsilon_1 \epsilon_2 T_1^4 - \epsilon_1 \epsilon_2 T_2^4]}{[1 - (1 - \epsilon_1)(1 - \epsilon_2)]}$$

$$[1 - (1 - \epsilon_1)(1 - \epsilon_2)]$$

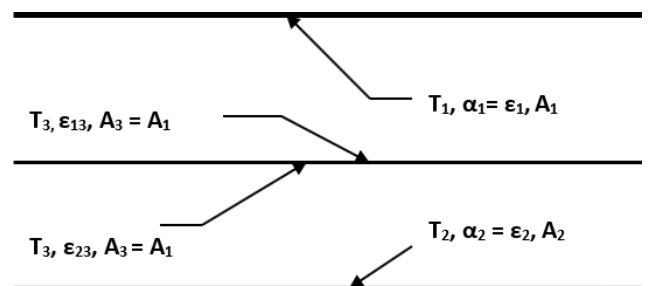
$$\sigma A_1 (T_1^4 - T_2^4)$$

$$\text{or } Q_{1-2} = \frac{\sigma A_1 (T_1^4 - T_2^4)}{[1/\epsilon_1 + 1/\epsilon_2 - 1]}$$

$$[1/\epsilon_1 + 1/\epsilon_2 - 1]$$

**Plane Radiation Shields:** It is possible to reduce the net radiation heat exchange between two infinite parallel gray surfaces by introducing a third surface in between them. If the third surface, known as the radiation shield is assumed to be very thin, then both sides of this surface can be assumed to be at the same temperature.

Figure shows a scheme for radiation heat exchange between two parallel infinite gray surfaces at two different temperatures  $T_1$  and  $T_2$  in presence of a radiation shield at a uniform temperature,  $T_3$ .



$$\text{Now } Q_{1-3} = \frac{\sigma (T_1^4 - T_3^4)}{[1/\epsilon_1 + 1/\epsilon_{13} - 1]}$$

$$Q_{3-2} = \frac{\sigma (T_3^4 - T_2^4)}{[1/\epsilon_{32} + 1/\epsilon_2 - 1]}$$

$$\text{And } Q_{1-3} = Q_{3-2}$$

For steady state conditions, these two must be equal. Therefore, we have

$$\frac{(T_1^4 - T_3^4)}{[1/\epsilon_1 + 1/\epsilon_{13} - 1]} = \frac{(T_3^4 - T_2^4)}{[1/\epsilon_{32} + 1/\epsilon_2 - 1]}$$

Let  $X = [1/\epsilon_1 + 1/\epsilon_{13} - 1]$

and  $Y = [1/\epsilon_{32} + 1/\epsilon_2 - 1]$

Then,

$$\frac{(T_1^4 - T_3^4)}{X} = \frac{(T_3^4 - T_2^4)}{Y}$$

Solving for  $T_3$  we get

$$T_3 = \left[ \frac{T_1^4 + (X/Y)T_2^4}{(1 + X/Y)} \right]^{1/4}$$

Substituting this value of  $T_3$  in above equation we get

$$Q_{1-3} / A_1 = Q_{3-2} / A_1 = (Q_{1-2} / A)_1 \text{ Rad.Shield} = \sigma \{ T_2^4 - [(T_1^4 + (X/Y)T_2^4)/(1 + X/Y)] \} / X$$

**Special case:**

When  $\epsilon_1 = \epsilon_2 = \epsilon_{13} = \epsilon_{32} = \epsilon$ , then  $X = Y = (2/\epsilon) - 1$

Hence,

$$T_3 = [(T_1^4 + T_2^4) / 2]^{1/4}$$

$$\sigma \{ T_1^4 - [(T_1^4 + T_2^4) / 2] \}$$

and  $[Q_{1-2} / A]_{1 \text{ rad shield}} = \frac{\sigma \{ T_1^4 - [(T_1^4 + T_2^4) / 2] \}}{2 [2/\epsilon - 1]}$

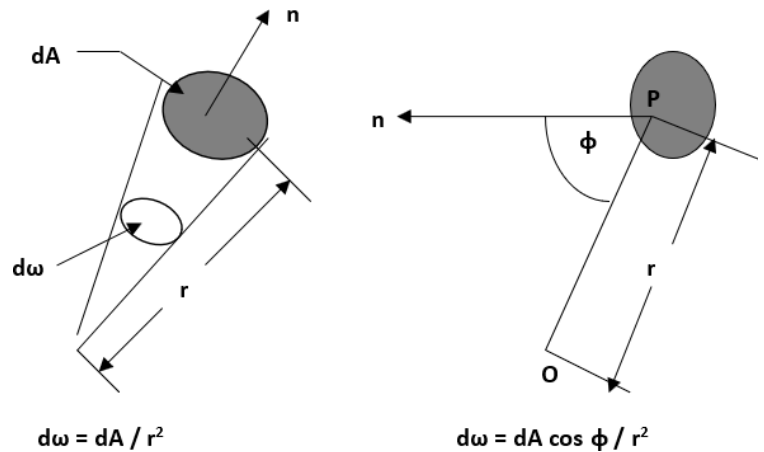
It can be seen from the above equation that when the emissivities of all surfaces are equal, the net radiation heat exchange between the surfaces in the presence of single radiation shield is 50% of the radiation heat exchange between the same two surfaces without the presence of a radiation shield. This statement can be generalised for N radiation shields as follows:

$$[Q_{1-2} / A]_{N \text{ shields}} = \frac{1}{(N + 1)} [Q_{1-2} / A]_{\text{without shield}}$$

**View Factor or Configuration Factor:** In engineering applications, we come across problems involving radiation heat exchange between two or more finite surfaces. When the surfaces are separated from each other by a non-participating medium that does not absorb, emit or reflect radiation, then the radiation heat exchange is not affected by the medium. A vacuum is a perfect non participating medium. However, air and many gases closely approximate this condition. For any two surfaces, the orientation of them with respect to each other affects the fraction of the radiation energy leaving one surface and striking the other directly. The concept of “View Factor” (also called as Configuration Factor/Shape Factor) has been utilised to formalise the effects of orientation in the radiation heat exchange between surfaces. Before the concept of view factor is introduced, two more terms have to be defined.

**Solid Angle and Intensity of Radiation:**

**Solid Angle:** The solid angle  $d\omega$  subtended by an elemental area  $dA$  surrounding point P with respect to any other point O in space is defined as the component of the area  $dA$  in the direction OP divided by the square of the distance between O and P. This is illustrated in figure. Solid angle is measured in Steradian (Sr).



Based on this definition, it can be readily inferred that the solid angle subtended by a hemispherical surface from its centre is  $2\pi$  ( $d\omega = 2\pi r^2/r^2$ ) and by a full glass sphere from its centre is  $4\pi$ .

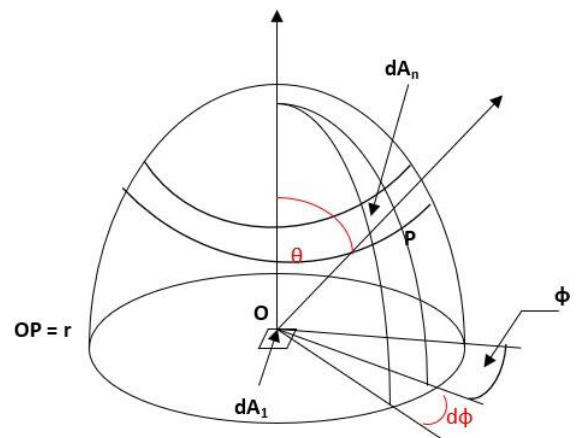
**Intensity of Radiation:** The total intensity of radiation emitted by the surface in a given direction is equal to the radiant flux passing in that direction per unit solid angle. If I is the intensity of radiation and E is the total emissive power, then by definition

$$I = dE/d\omega$$

$$E = \int I d\omega$$

where the integration is carried out over all directions encompassed by a hemisphere.

Consider an elemental area  $dA_1$  whose total emissive power is  $E_1$ . This total radiant energy emitted by  $dA_1$  can be intercepted by a hemisphere as shown in figure.



If  $I$  is the intensity of radiation at any point  $P$  on the surface of the hemisphere due to emission by an elemental area  $dA_1$  at  $O$ , then

$$E_1 = \int I_1 \cos\theta \, dw = \int I_1 \cos\theta \left( \frac{dA_n}{r^2} \right)$$

$$E_1 = \int_{\theta=0}^{\frac{\pi}{2}} \int_{\phi=0}^{2\pi} \left[ \frac{I_1 \cos\theta \times r \sin\theta \, d\theta \times r \, d\phi}{r^2} \right]$$

Assuming that  $I_1$  is same in all directions (Lambert's Law)

$$E_1 = I_1 \int_{\theta=0}^{\frac{\pi}{2}} \int_{\phi=0}^{2\pi} \cos\theta \sin\theta \, d\theta \, d\phi$$

$$E_1 = \pi I_1$$

If the surface is a black surface then  $E_b = \pi I_b$

**View Factor Between Two Elemental Surfaces:**

Consider 2 elemental surfaces of area  $dA_1$  and  $dA_2$  as shown in figure. Let their normals  $n_1$  and  $n_2$  make angles  $\phi_1$  and  $\phi_2$  with the line joining the centroid of the two elemental areas. Let  $dw_2$  be the solid angle subtended by  $dA_2$  at  $dA_1$  and  $dw_1$  be the solid angle subtended by  $dA_1$  at  $dA_2$ .

Let  $I_1$  be the intensity of radiation from  $dA_1$  striking  $dA_2$  and  $I_2$  be the intensity of radiation from  $dA_2$  striking  $dA_1$ . If  $dq_{1 \rightarrow 2}$  is the radiant energy leaving  $dA_1$  and striking  $dA_2$  then

$$dq_{1 \rightarrow 2} = I_1 dA_1 \cos\phi_1 \, dw_2$$

Or  $dq_{1 \rightarrow 2} = I_1 dA_1 \cos\phi_1 \frac{dA_2 \cos\phi_2}{r^2}$

Radiation energy leaving  $dA_1 = dq_{r1} = E_1 dA_1$

Fraction of energy leaving  $dA_1$  and striking  $dA_2$  is defined as the view factor of  $dA_2$  with respect to  $dA_1$  and is denoted by  $dF_{1-2}$

Therefore  $dF_{1-2} = \frac{dq_{1-2}}{dq_{r1}} = \frac{I_1 dA_1 dA_2 \cos\phi_1 \cos\phi_2}{r^2 E_1 dA_1}$

Using the relation  $E_1 = \pi I_1$  we have

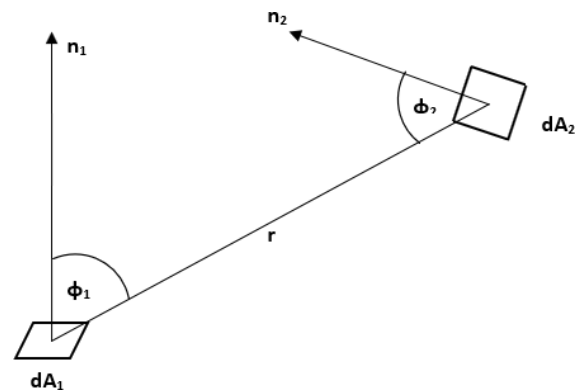
$$dF_{1-2} = \frac{dA_2 \cos\phi_1 \cos\phi_2}{\pi r^2}$$

Similarly, the view factor of  $dA_1$  with respect to  $dA_2$  is denoted by  $dF_{2-1}$  and given by

$$dF_{2-1} = \frac{dA_1 \cos\phi_1 \cos\phi_2}{\pi r^2}$$

It follows from the above equations that

$$dA_1 dF_{1-2} = dA_2 dF_{2-1}$$

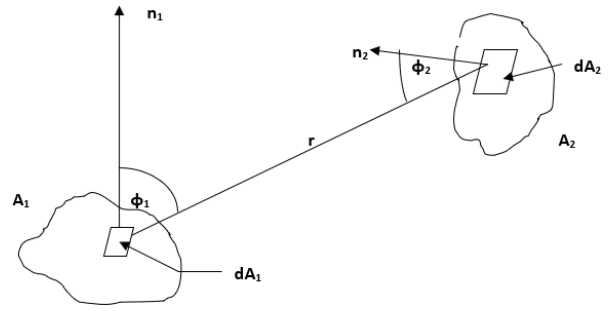


**View Factor for Finite Surfaces:**

Consider two finite surfaces of areas  $A_1$  and  $A_2$  as shown in Fig 10.12. If  $n_1$  and  $n_2$  are the normals for elemental areas  $dA_1$  and  $dA_2$  then energy leaving  $dA_1$  and reaching  $dA_2$  is given by

$$dq_{1 \rightarrow 2} = I_1 dA_1 \cos \phi_1 dA_2 \cos \phi_2 / r^2$$

Hence  $Q_{1 \rightarrow 2} = \int \int_{A_1 A_2} \{ I_1 dA_1 dA_2 \cos \phi_2 / r^2 \}$ .



Total radiation emitted by  $A_1 = Q_{r1} = E_1 A_1 = \pi I_1 A_1$

Fraction of energy which leaves  $A_1$  and reaches  $A_2$  is given by

$$F_{1 \rightarrow 2} = \frac{Q_{1 \rightarrow 2}}{Q_{r1}} = \frac{\int \int \{ I_1 dA_1 dA_2 \cos \phi_2 / r^2 \}}{\pi I_1 A_1}$$

Or 
$$F_{1 \rightarrow 2} = \frac{1}{A_1} \int \int \{ I_1 dA_1 dA_2 \cos \phi_2 / (\pi r^2) \}$$

Similarly 
$$F_{2 \rightarrow 1} = \frac{1}{A_2} \int \int \{ I_1 dA_1 dA_2 \cos \phi_2 / (\pi r^2) \}$$

It follows from above equations

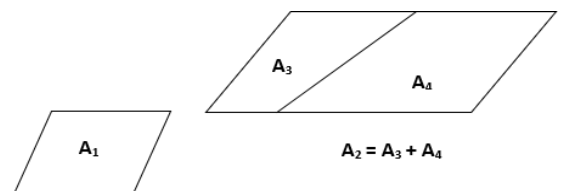
$$A_1 F_{1 \rightarrow 2} = A_2 F_{2 \rightarrow 1}$$

**Properties of view factor:** Consider an enclosure consisting of  $N$  zones, each of surface area  $A_i$  ( $i = 1, 2, 3 \dots N$ ). The surface of each zone may be plane, convex or concave. For the enclosure, the following relations hold good.

1.  $A_i F_{i \rightarrow j} = A_j F_{j \rightarrow i}$ ,  $i = 1, 2, 3 \dots N$ ,  
 $j = 1, 2, 3 \dots N$
2.  $F_{i \rightarrow i} = 0$  if  $A_i$  is plane or convex (i.e.  $A_i$  cannot see itself)  
\* 0 if  $A_i$  is concave.
3.  $F_{1 \rightarrow 1} + F_{1 \rightarrow 2} + \dots + F_{1 \rightarrow N} = 1$   
 $F_{2 \rightarrow 1} + F_{2 \rightarrow 2} + \dots + F_{2 \rightarrow N} = 1$   
|  
|  
 $F_{N \rightarrow 1} + F_{N \rightarrow 2} + \dots + F_{N \rightarrow N} = 1$

In short,  $\sum_{j=1}^N F_{i \rightarrow j} = 1$ ,  $i = 1, 2, 3 \dots N$

4. When there are two surfaces, one surface say  $A_1$  is completely enclosed by  $A_2$  and if  $A_1$  cannot see itself then,  $F_{1 \rightarrow 2} = 1$  and  $F_{2 \rightarrow 1} = A_1/A_2$
5. The view factor  $F_{1 \rightarrow 2}$  between surfaces  $A_1$  and  $A_2$  (figure) is equal to the sum of the view



factors  $F_{1-3}$  and  $F_{1-4}$  if the two areas  $A_3$  and  $A_4$  together make up the area  $A_2$ .

i.e.,  $F_{1-2} = F_{1-3} + F_{1-4}$ . It should however be noted that  $F_{2-1} \neq F_{3-1} + F_{4-1}$

**View factors for standard configurations:** The determination of view factors has been the object of considerable research. In cases where the integrals in above equations cannot be solved analytically, numerical methods have been used. Some of these results are represented graphically for certain standard configurations like

- (i) Shape factors between parallel rectangles of equal size.
- (ii) Shape factors between rectangles perpendicular to each other and having a common edge
- (iii) Shape factor from an elemental area  $dA_1$  to a rectangular area  $A_2$
- (iv) Shape factor between two coaxial parallel discs
- (v) Shape factors for concentric cylinders of finite length etc.

With the help of those charts and View Factor algebra, shape factors between surfaces not covered above can be determined.

## Module 5: HEAT EXCHANGERS

Heat exchangers are devices used to transfer heat energy from one fluid to another. Typical heat exchangers experienced by us in our daily lives include condensers and evaporators used in air conditioning units and refrigerators. Boilers and condensers in thermal power plants are examples of large industrial heat exchangers. There are heat exchangers in our automobiles in the form of radiators and oil coolers. Heat exchangers are also abundant in chemical and process industries.

There is a wide variety of heat exchangers for diverse kinds of uses, hence the construction also would differ widely. However, in spite of the variety, most heat exchangers can be classified into some common types based on some fundamental design concepts. We will consider only the more common types here for discussing some analysis and design methodologies.

Some typical examples of heat exchanger applications are:

1. Thermal power plants (boilers, superheaters, steam condensers, etc.)
2. Refrigeration and air-conditioning (evaporators, condensers, coolers)
3. Automobile industry (radiators, all engine cooling and fuel cooling arrangements)
4. Chemical process industry (variety of heat exchangers between different types of fluids, in combustors and reactors)
5. Cryogenic industry (condenser-reboilers used in distillation columns, evaporators to produce gas from cryogenic liquids, etc.)
6. Research ('regenerators' used in Stirling engines, special ceramic heat exchangers used in ultra-low temperature devices, superconducting magnet systems, etc.).

### Types of Heat Exchangers:

Heat exchangers may be classified in several ways:

- (i) according to heat exchange process
- (ii) according to relative direction of flow of hot and cold fluids
- (iii) according to constructional features, compactness, etc.
- (iv) according to the state of the fluid in the heat exchanger.

#### 1. According to Heat Exchange Process

- Direct Contact – Two immiscible fluids come in direct contact with each other and exchange heat.

Ex: Air – Water exchanging heat in a Cooling Tower

- Indirect Contact – No direct contact between two fluids during heat transfer process.

- Regenerators – Hot and Cold fluids flow alternatively through a solid matrix.

Ex: Rotary regenerator

- Recuperators – Heat transfer between two fluids could be carried out by transmission through a separating wall.

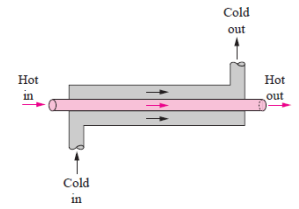
Ex: Automobile radiators, Oil coolers, Boilers, etc.,



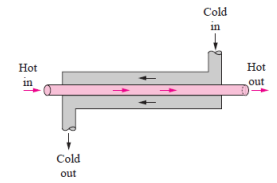


2. According to relative direction of fluid flow

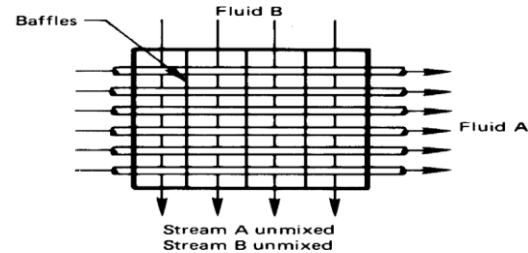
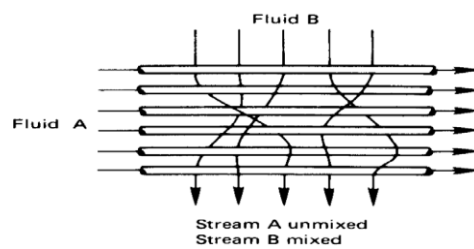
- Parallel Flow – If the hot and cold fluids flow parallel to each other, it is known as ‘parallel flow’ heat exchanger.



- Counter Flow – If the hot and cold fluids flow opposite to each other, it is of ‘counter-flow’ type.



- Cross Flow – If the fluids flow perpendicular to each other, then, we have ‘cross flow’ type of heat exchanger.

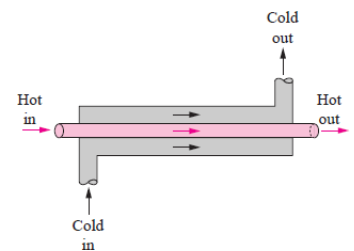


Further, when a fluid is constrained to flow within a channel (such as a tube), the fluid is said to be ‘unmixed’; otherwise, it is ‘mixed’. In the left figure, fluid A is unmixed since it flows constrained within the tubes, whereas fluid B is perfectly mixed as it flows through the heat exchanger. In the right figure, both fluids are constrained to flow within the tubes and therefore, both the fluids are unmixed.

3. According to Design and Constructional Features

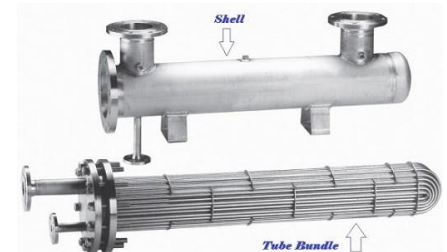
- Concentric Tube Heat exchanger

In concentric tubes type of heat exchanger, one tube is located inside another; one fluid flows through the inside tube and the other fluid flows in the annular space between the tubes.



- Shell and Tube Heat exchanger

Shell and tube type of heat exchanger is very popular in industry because of its reliability and high heat transfer effectiveness. Here, one of the fluids flows within a bundle of tubes placed within a shell. And, the other fluid flows through the shell over the surfaces of the tubes. Suitable baffles are provided within the shell to make the shell fluid change directions and provide good turbulence, so that heat transfer coefficient is increased.



- Compact Heat exchanger

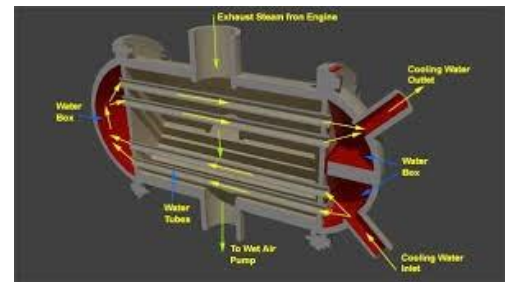
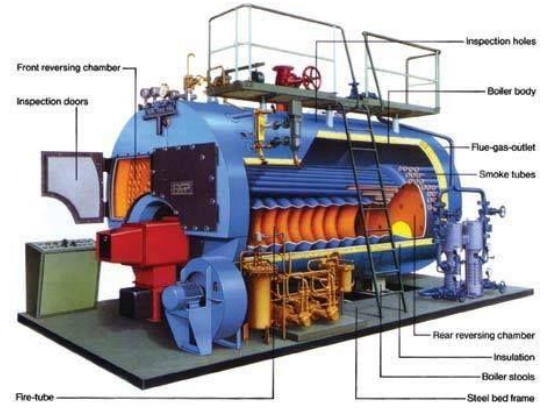
Compact heat exchangers are special purpose heat exchangers which provide very high surface area per cubic metre of volume, known as ‘area density’. According to

usually accepted norms, a 'compact heat exchanger' has an area density of  $700 \text{ m}^2/\text{m}^3$  or more. These are generally used for gases, since usually gas side heat transfer coefficient is small and therefore, it is needed to provide larger areas. Compact heat exchangers are of plate-fin type or tube-fin type.



4. According to the state of fluid in heat exchanger

- Evaporator – If, one of the fluids evaporates in a heat exchanger, temperature of this fluid will remain constant throughout the length of heat exchanger, whereas the temperature of the other fluid, which supplies the latent heat of evaporation to the evaporating fluid, goes on decreasing along the length of the heat exchanger. Such a heat exchanger is called an 'Evaporator'.
- Condenser –A heat exchanger may be used to condense a fluid in which case the condensing fluid will be at a constant temperature throughout the length of the heat exchanger, while the other (cold) fluid will increase in temperature as it passes through the heat exchanger, absorbing the latent heat of condensation released by the condensing fluid. Such a heat exchanger is called a 'Condenser'.



**Overall Heat Transfer Coefficient:** In most of the practical cases of heat exchangers, temperature of the hot fluid ( $T_a$ ) and that of the cold fluid ( $T_b$ ) are known; then we would like to have the heat transfer given by a simple relation of the form

$$Q = U \times A \times (T_a - T_b) = U \times A \times \Delta T$$

where,  $Q$  is the heat transfer rate (W),  $A$  is the area of heat transfer perpendicular to the direction of heat transfer, and  $(T_a - T_b) = \Delta T$  is the overall temperature difference between the temperature of hot fluid ( $T_a$ ) and that of the cold fluid ( $T_b$ ).

In a normally used recuperative type of heat exchanger, the hot and cold fluids are separated by a solid wall. This may be a flat type of wall (as in the case of plate-fin type of heat exchangers), or, more often, a cylindrical wall (as in the case of a tube-in-tube type of heat exchangers). See figure. In general, the overall heat transfer coefficient is related to the total thermal resistance of the system, as follows:

$$U = \frac{1}{A \cdot \sum R_{th}} \quad \text{W/m}^2\text{K}$$

Therefore, the task of finding the overall heat transfer coefficient reduces to finding out the total thermal resistance of the system.

**For cylindrical wall:**

Remember that for a cylindrical wall, thermal resistance is:

$$R_{th} = \frac{\ln\left(\frac{r_o}{r_i}\right)}{2 \cdot \pi \cdot k \cdot L}$$

and, convective resistance is  $1/(h \cdot A)$  and the resistances are in series.

However, the area to be considered has to be specified since the inner surface area and the outer surface area of the cylinder are different.

Now, we have, the general relation for U:

$$U = \frac{1}{A \cdot \sum R_{th}} \quad \text{WW} \quad (m^2K)$$

i.e.

$$U \cdot A = \frac{1}{\sum R_{th}}$$

We can also write:

$$U_{ii} \cdot A_{ii} = U_o \cdot A_o = \frac{1}{\sum R_{th}}$$

Therefore, referred to outer surface area, U becomes:

$$U_o \cdot A_o = \frac{1}{\frac{1}{h_{ii} A_{ii}} + \frac{\ln\left(\frac{r_o}{r_i}\right)}{2 \cdot \pi \cdot k \cdot L} + \frac{1}{h_o A_o}}$$

Now, for a cylindrical system, we have:

$$A_{ii} = 2 \times \pi \times r_i \times L$$

and,

$$A_o = 2 \times \pi \times r_o \times L$$

Then,

$$U_o = \frac{1}{\frac{1}{h_{ii} A_{ii}} + \frac{\ln\left(\frac{r_o}{r_i}\right)}{2 \cdot \pi \cdot k \cdot L} + \frac{1}{h_o A_o}}$$

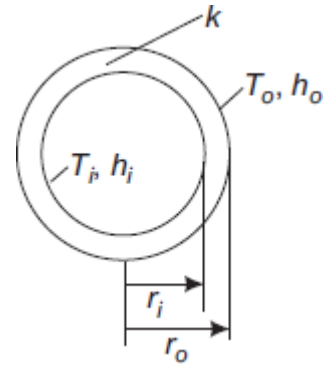
$$U_o = \frac{1}{\frac{1}{h_{ii} \left(\frac{r_o}{r_i}\right)} + \frac{\ln\left(\frac{r_o}{r_i}\right)}{k} + \frac{1}{h_o}}$$

Similarly, referred to inner surface area, U becomes:

$$U_o = \frac{1}{\frac{1}{h_{ii}} + \frac{\ln\left(\frac{r_o}{r_i}\right)}{k} + \frac{1}{h_o \left(\frac{r_i}{r_o}\right)}}$$

Again, if the thermal resistance of the wall is negligible compared to other resistances, (i.e. high value of thermal conductivity, k), or, wall thickness of the tube is very small (i.e.  $(r_i/r_o) \gg 1$ ), we get:

$$U = \frac{1}{\frac{1}{h_{ii}} + \frac{1}{h_o}} \quad \text{WW} \quad (m^2K)$$



**Fouling factors:** Note that above analysis was for clean heat transfer surfaces. However, with passage of time, the surfaces become 'dirty' because of scaling, deposits, corrosion, etc. This results in a reduction in heat transfer coefficient since the scale offers a thermal resistance to heat transfer.

Fouling may be categorized as follows:

- (i) due to scaling or precipitation
- (ii) due to deposits of finely divided particulates
- (iii) due to chemical reaction
- (iv) due to corrosion
- (v) due to attachments of algae or other biological materials
- (vi) due to crystallization on the surface by subcooling.

Effect of fouling is accounted for by a term called, 'Fouling factor', (or, 'dirt factor'), defined as:

$$R_{ff} = \frac{1}{U_{dirty}} - \frac{1}{U_{clean}} \quad m^2K/W$$

$R_f$  is zero for a new heat exchanger.  $R_f$  for a fouled heat exchanger cannot be 'calculated' theoretically, but has to be determined experimentally by finding out the heat transfer coefficients for a 'clean' heat exchanger and a 'dirty' heat exchanger of identical design, operating under identical conditions.

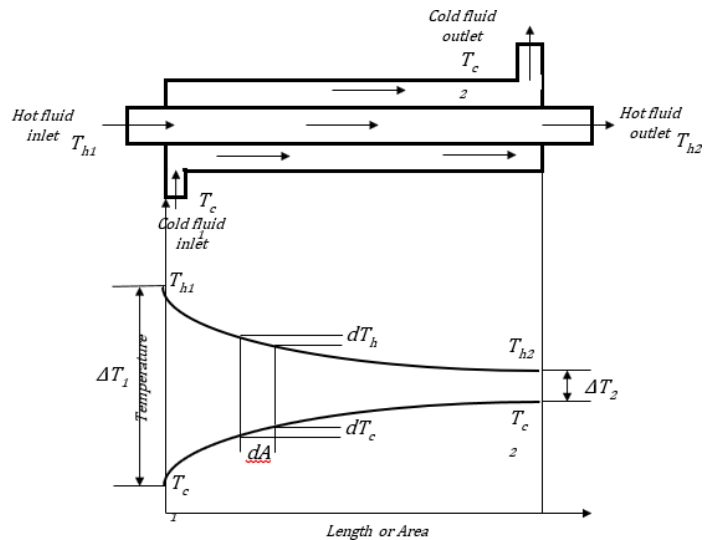
While taking into account the effect of fouling, the 'fouling resistance' (=  $R_f$ /area) should be added to the other thermal resistances. For example, for a tube, we can write:

$$U_{ii} \cdot A_{ii} = U_o \cdot A_o = \frac{1}{\sum R_{th}} = \frac{1}{\frac{1}{h_{ii} A_{ii}} + \frac{R_{ffi}}{A_{ii}} + \frac{\ln \left( \frac{r_o}{r_{ii}} \right)}{2 \cdot \pi \cdot k \cdot L} + \frac{R_{ffo}}{A_o} + \frac{1}{h_o A_o}}$$

where,  $R_{fi}$  and  $R_{fo}$  are the fouling factors for the inside and outside surfaces, respectively, and  $L$  is the length of tube. Fouling factor depends on flow velocity and operating temperature; fouling increases with decreasing velocity and increasing temperature.

**LMTD Method for Heat Exchanger Analysis**

**Parallel Flow Heat Exchanger:** Consider a double pipe, parallel flow heat exchanger, in which a hot fluid and a cold fluid flow parallel to each other, separated by a solid wall. Hot fluid enters at a temperature of  $T_{h1}$  and leaves the heat exchanger at a temperature of  $T_{h2}$ ; cold fluid enters the heat exchanger at a temperature of  $T_{c1}$  and leaves at a temperature of  $T_{c2}$ . This situation is shown in figure.



We desire to get an expression for the rate of heat transfer in this heat exchanger in the following form:

$$Q = UA(\Delta T)_{mean} \text{ ----- (a)}$$

U = Overall Heat transfer coefficient

A = Area of heat transfer

$(\Delta T)_{mean}$  = Mean temperature difference between fluids.

Now, we make the following assumptions:

- i. U is considered constant throughout the length of the heat exchanger
- ii. Properties of fluids are considered constant.
- iii. Heat exchange takes place only between the two fluids.
- iv. Changes in potential and kinetic energy are negligible.
- v. Temperature of both fluids remain constant over a given cross section.

Area 'A' is constant for a given heat exchanger. However, we see from figure that the temperature of the two fluids vary along the length (or area) of the heat exchanger, i.e. the temperature difference between the hot and cold fluids is not a constant along the length of the heat exchanger, but varies along the length. Our aim is to find out the appropriate 'mean temperature difference ( $\Delta T_m$ )' between the hot and cold fluids, so that Eq. (a) can be applied. We proceed as follows:

Consider an elemental area  $dA$ , applying 1<sup>st</sup> law of thermodynamics,

Heat given up by hot fluid = Heat received by the cold fluid

$$\text{i.e. } dQ_h = -m_h C_{ph} dT_h = m_c C_{pc} dT_c \text{ ----- (b)}$$

$$dT_h = \frac{-dQ}{m_h C_{ph}} \text{ and } dT_c = \frac{dQ}{m_c C_{pc}}$$

where,  $m_h$  and  $m_c$  are the mass flow rates, and  $C_{ph}$  and  $C_{pc}$  are the specific heats of hot and cold fluids, respectively. Therefore,

$$dT_h - dT_c = d(T_h - T_c) = -dQ \left[ \frac{1}{m_h C_{ph}} + \frac{1}{m_c C_{pc}} \right]$$

Substituting for  $dQ$  from Eq. (b), we get:

$$\begin{aligned} d(T_h - T_c) &= -U(T_h - T_c)dA \left[ \frac{1}{m_h C_{ph}} + \frac{1}{m_c C_{pc}} \right] \\ \frac{d(T_h - T_c)}{(T_h - T_c)} &= -U \left[ \frac{1}{m_h C_{ph}} + \frac{1}{m_c C_{pc}} \right] dA \end{aligned}$$

Integrating between inlet and exit of the heat exchanger:

$$\ln \left[ \frac{T_{h2} - T_{c2}}{T_{h1} - T_{c1}} \right] = -UA \left[ \frac{1}{m_h C_{ph}} + \frac{1}{m_c C_{pc}} \right]$$

Now considering the total heat transfer rate for the entire heat exchanger, we have,

$$m_h C p_h = \frac{dd}{T_{h1}-T_{h2}} \quad \text{and} \quad m_c C p_c = \frac{dd}{T_{c2}-T_{c1}}$$

Substituting in the above equation,

$$\ln \frac{T_{h2} - T_{c2}}{T_{h1} - T_{c1}} = \frac{-UA}{\frac{m_h C p_h (T_{h1} - T_{h2}) + m_c C p_c (T_{c2} - T_{c1})}{m_h C p_h (T_{h1} - T_{h2})}}$$

i.e.,

$$Q = \frac{UA (m_h C p_h (T_{h1} - T_{h2}) + m_c C p_c (T_{c2} - T_{c1}))}{\ln \frac{T_{h2} - T_{c2}}{T_{h1} - T_{c1}}}$$

or,

$$Q = UA \Delta T_m$$

$$\text{Where, } \Delta T_m = \frac{(T_{h2} - T_{c2}) - (T_{h1} - T_{c1})}{\ln \frac{T_{h2} - T_{c2}}{T_{h1} - T_{c1}}}$$

Since this mean temperature difference varies in a logarithmic manner, it is called 'Logarithmic Mean Temperature Difference' or, simply LMTD.

or,

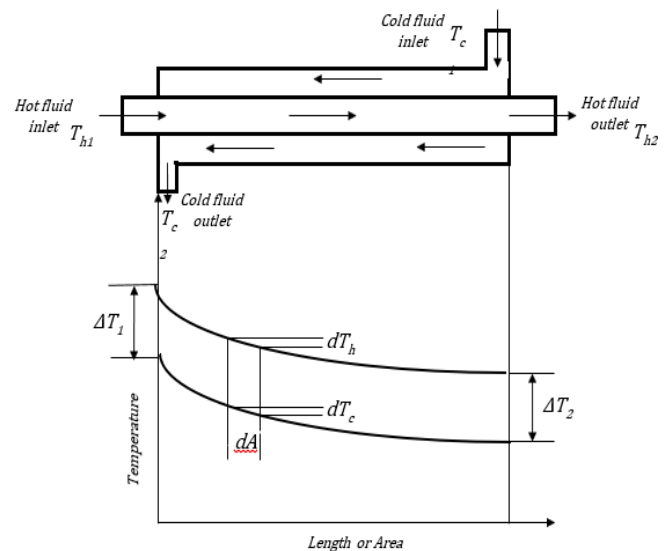
$$\Delta T_m = \frac{\Delta T_2 - \Delta T_1}{\ln \frac{\Delta T_2}{\Delta T_1}}$$

Note: For special case when  $\Delta T_1 = \Delta T_2$ ,  $\Delta T_m = \frac{0}{0}$  which is indeterminate.

In this case, by applying L Hospital's rule it can be proved that,

$$\Delta T_m = \Delta T_1 = \Delta T_2$$

**Counter Flow Heat Exchanger:** Consider a double pipe, counter flow heat exchanger, in which a hot fluid and a cold fluid flow parallel to each other, separated by a solid wall. Hot fluid enters at a temperature of  $T_{h1}$  and leaves the heat exchanger at a temperature of  $T_{h2}$ ; cold fluid enters the heat exchanger at a temperature of  $T_{c1}$  and leaves at a temperature of  $T_{c2}$ . This situation is shown in figure.



We desire to get an expression for the rate of heat transfer in this heat exchanger in the following form:

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U = Overall Heat transfer coefficient

A = Area of heat transfer

$(\Delta T)_{mean}$  = Mean temperature difference between fluids.

Now, we make the following assumptions:

- i. U is considered constant throughout the length of the heat exchanger
- ii. Properties of fluids are considered constant.
- iii. Heat exchange takes place only between the two fluids.
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Area 'A' is constant for a given heat exchanger. However, we see from figure that the temperature of the two fluids vary along the length (or area) of the heat exchanger, i.e. the temperature difference between the hot and cold fluids is not a constant along the length of the heat exchanger, but varies along the length. Our aim is to find out the appropriate 'mean temperature

difference  $(\Delta T_m)$  between the hot and cold fluids, so that Eq. (a) can be applied. We proceed as follows:

Consider an elemental area  $dA$ , applying 1<sup>st</sup> law of thermodynamics,  
*Heat given up by hot fluid = Heat received by the cold fluid*

$$\text{ii. e. } dQ = -m_h C_{p_h} dT_h = m_c C_{p_c} dT_c \text{----- (b)}$$

$$dT_h = \frac{-dd}{m_h C_{p_h}} \text{ and } dT_c = \frac{dd}{m_c C_{p_c}}$$

where,  $m_h$  and  $m_c$  are the mass flow rates, and  $C_{p_h}$  and  $C_{p_c}$  are the specific heats of hot and cold fluids, respectively. Therefore,

$$dT_h - dT_c = d(T_h - T_c) = -dQ \left( \frac{1}{m_h C_{p_h}} - \frac{1}{m_c C_{p_c}} \right)$$

Substituting for  $dQ$  from Eq. (b), we get:

$$d(T_h - T_c) = -U(T_h - T_c)dA \left( \frac{1}{m_h C_{p_h}} - \frac{1}{m_c C_{p_c}} \right)$$

$$\frac{d(T_h - T_c)}{(T_h - T_c)} = -U \left( \frac{1}{m_h C_{p_h}} - \frac{1}{m_c C_{p_c}} \right) dA$$

Integrating between inlet and exit of the heat exchanger:

$$\ln \frac{T_{h_2} - T_{c_2}}{T_{h_1} - T_{c_1}} = -UA \left( \frac{1}{m_h C_{p_h}} - \frac{1}{m_c C_{p_c}} \right)$$

Now considering the total heat transfer rate for the entire heat exchanger, we have,

$$m_h C_{p_h} = \frac{dQ}{T_{h_1} - T_{h_2}} \text{ and } m_c C_{p_c} = \frac{dQ}{T_{c_2} - T_{c_1}}$$

Substituting in the above equation,

$$\ln \frac{T_{h_2} - T_{c_2}}{T_{h_1} - T_{c_1}} = \frac{-UA}{Q} (T_{h_1} - T_{h_2} - T_{c_2} + T_{c_1})$$

$$Q = \frac{UA(T_{h_2} - T_{c_1} - T_{h_1} + T_{c_2})}{\ln \frac{T_{h_2} - T_{c_1}}{T_{h_1} - T_{c_2}}}$$

i.e.,

or,

$$Q = UA \Delta T_m$$

$$\text{Where, } \Delta T_m = \frac{T_{h_2} - T_{c_1} - T_{h_1} + T_{c_2}}{\ln \frac{T_{h_2} - T_{c_1}}{T_{h_1} - T_{c_2}}}$$

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Note: For special case when  $\Delta T_1 = \Delta T_2$ ,  $\Delta T_m = \frac{0}{0}$  which is indeterminate.

In this case, by applying L Hospital's rule it can be proved that,

$$\Delta T_m = \Delta T_1 = \Delta T_2$$

**Correction Factors for Multi-pass and Cross-flow Heat Exchangers:**

LMTD relations derived above are applicable to parallel flow and counter-flow heat exchangers only. But, in practice, cross-flow heat exchangers (e.g. automobile radiators) and shell-and-tube heat exchangers, with more than one pass in shell side and/or tube side, are also used. In such cases, the flow situation is complex and the analytic relations for mean temperature difference are very complicated. Then, first, LMTD is calculated as if for a counter-flow heat exchanger with the inlet and exit temperatures for the two fluids as per the actual data, and next, a 'correction factor ( $F$ )' is applied to the calculated LMTD to get the mean temperature difference between the fluids. Now, heat transfer rate is calculated as:

$$Q = UA(F \times LMTD), W$$

where,  $A$  is the area of heat transfer,  $U$  is the overall heat transfer coefficient referred to that area, and  $F$  is the correction factor. **Note again that LMTD is calculated as if for a counter-flow heat exchanger, taking the inlet and exit temperatures of the two fluids the same as for the actual heat exchanger.**



## Effectiveness – NTU Method for Heat Exchanger Analysis

**Effectiveness of a heat exchanger ( $\epsilon$ ):**

$$\epsilon = \frac{Q_{act}}{Q_{max}}$$

where,  $Q$  = actual heat transferred in the heat exchanger

$Q_{max}$  = maximum possible heat transfer in the heat exchanger

Now, actual heat transfer rate in a heat exchanger is given by:

$$Q = m_h c_{ph} (T_{h1} - T_{h2}) = C_h (T_{h1} - T_{h2})$$

and,

$$Q = m_c c_{pc} (T_{c2} - T_{c1}) = C_c (T_{c2} - T_{c1})$$

where,  $C_h$  = capacity rate of the hot fluid, and

$C_c$  = capacity rate of the cold fluid

Now,  $C_h$  may be equal to  $C_c$  or less than  $C_c$  or greater than  $C_c$ .

If  $C_h < C_c$ , we designate  $C_h$  as  $C_{min}$ ;

Instead, if  $C_h > C_c$ , we designate  $C_c$  as  $C_{min}$ .

And, in each case, capacity rate of the other fluid is designated as  $C_{max}$ .

**Capacity Ratio (C):**

Capacity ratio is defined as:

$$C = \frac{C_{min}}{C_{max}}$$

**Number of Transfer Units (NTU):**

Number of Transfer Units (which is a dimensionless number), is defined as:

$$NUT = \frac{UA}{C_{min}}$$

where,  $U$  is the overall heat transfer coefficient and  $A$  is the corresponding heat transfer area. For given value of  $A$  and flow conditions, NTU is a measure of the area (i.e. size) of the heat exchanger. Larger the NTU, larger the size of the heat exchanger.

**Maximum possible heat transfer in a heat exchanger ( $Q_{max}$ ):**

Now, consider a heat exchanger where the hot fluid is cooled from a temperature of  $T_{h1}$  to  $T_{h2}$  and the cold fluid heated from  $T_{c1}$  to  $T_{c2}$ . So, the maximum temperature differential in the heat exchanger is  $(T_{h1} - T_{c1})$ . Now, if the heat exchanger had an infinite area, the hot fluid will be cooled from  $T_{h1}$  to  $T_{c1}$  or the cold fluid may be heated from  $T_{c1}$  to  $T_{h1}$ . However, which fluid will experience the maximum temperature differential  $(T_{h1} - T_{c1})$  will depend upon which fluid has the minimum capacity rate.

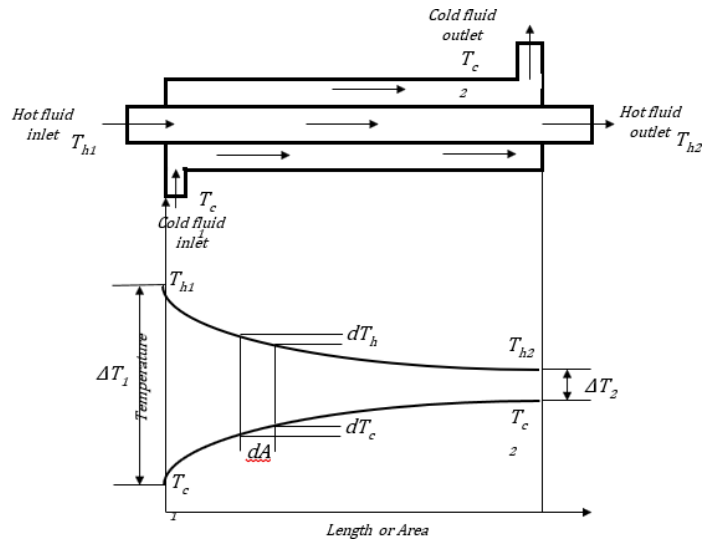
If hot fluid has the minimum capacity rate, we can write:

$$Q_{max} = C_h (T_{h1} - T_{c1}) \text{ iff } C_h \text{ is minimum capacity, } C_{min}$$

Instead, if cold fluid has the minimum capacity rate, we write:

$$Q_{max} = C_c (T_{h1} - T_{c1}) \text{ iff } C_c \text{ is minimum capacity, } C_{min}$$

**Parallel Flow Heat Exchanger:** Consider a double pipe, parallel flow heat exchanger, in which a hot fluid and a cold fluid flow parallel to each other, separated by a solid wall. Hot fluid enters at a temperature of  $T_{h1}$  and leaves the heat exchanger at a temperature of  $T_{h2}$ ; cold fluid enters the heat exchanger at a temperature of  $T_{c1}$  and leaves at a temperature of  $T_{c2}$ . This situation is shown in figure.



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Consider an elemental area  $dA$ , applying 1<sup>st</sup> law of thermodynamics,

Heat given up by hot fluid = Heat received by the cold fluid

$$-m_h C_{ph} dT_h = m_c C_{pc} dT_c \text{ ----- (b)}$$

$$dT_h = \frac{-dd}{m_h C_{ph}} \text{ and } dT_c = \frac{dd}{m_c C_{pc}}$$

where,  $m_h$  and  $m_c$  are the mass flow rates, and  $C_{ph}$  and  $C_{pc}$  are the specific heats of hot and cold fluids, respectively. Therefore,

$$dT_h - dT_c = d(T_h - T_c) = -dQ \left[ \frac{1}{m_h C_{ph}} + \frac{1}{m_c C_{pc}} \right]$$

Substituting for  $dQ$  from Eq. (b), we get:

$$d(T_h - T_c) = -U(T_h - T_c) dA \left[ \frac{1}{m_h C_{ph}} + \frac{1}{m_c C_{pc}} \right]$$

$$\frac{d(T_h - T_c)}{(T_h - T_c)} = -U \left[ \frac{1}{m_h C_{ph}} + \frac{1}{m_c C_{pc}} \right] dA$$

Integrating between inlet and exit of the heat exchanger:

$$\ln \left[ \frac{T_{h2} - T_{c2}}{T_{h1} - T_{c1}} \right] = -UA \left[ \frac{1}{m_h C_{ph}} + \frac{1}{m_c C_{pc}} \right]$$

Now, out of the two fluids, one is the 'minimum' fluid and the other is the 'maximum' fluid.

Whichever may be the minimum fluid, we can write the above equation as:

$$\ln \frac{T_{h2} - T_{c2}}{T_{h1} - T_{c1}} = -UA \left( \frac{1}{C_{min}} + \frac{1}{C_{max}} \right)$$

$$= -\frac{UA}{C_{min}} \left( 1 + \frac{C_{min}}{C_{max}} \right)$$

$$\frac{T_{h2} - T_{c2}}{T_{h1} - T_{c1}} = e^{-\frac{UA}{C_{min}} \left( 1 + \frac{C_{min}}{C_{max}} \right)}$$

Now we know that,

$$\epsilon \epsilon = \frac{dd_{act}}{dd_{max}} = \frac{C_h (T_{h1} - T_{h2})}{C_{min} (T_{h1} - T_{c1})} = \frac{C_h (T_{c2} - T_{c1})}{C_{min} (T_{h1} - T_{c1})}$$

$$\Rightarrow T_{h2} = T_{h1} - \epsilon \epsilon \frac{C_{min}}{C_h} (T_{h1} - T_{c1})$$

$$T_{c2} = T_{c1} + \epsilon \epsilon \frac{C_{min}}{C_c} (T_{h1} - T_{c1})$$

Now substituting the values of  $T_{h2}$  and  $T_{c2}$  in the equation

$$\frac{T_{h2} - T_{c2}}{T_{h1} - T_{c1}} = e^{-\frac{UA}{C_{min}} \left( 1 + \frac{C_{min}}{C_{max}} \right)}$$

$$\frac{T_{h1} - \epsilon \epsilon \frac{C_{min}}{C_h} (T_{h1} - T_{c1}) - T_{c1} + \epsilon \epsilon \frac{C_{min}}{C_c} (T_{h1} - T_{c1})}{T_{h1} - T_{c1}} = e^{-\frac{UA}{C_{min}} \left( 1 + \frac{C_{min}}{C_{max}} \right)}$$

Now let,  $NTU = \frac{UA}{C_{min}}$

$$\frac{T_{h1} - T_{c1} \left( \epsilon \epsilon \frac{C_{min}}{C_h} - \epsilon \epsilon \frac{C_{min}}{C_c} + 1 \right)}{T_{h1} - T_{c1}} = e^{-NTU \left( 1 + \frac{C_{min}}{C_{max}} \right)}$$

$$1 - \epsilon \epsilon \frac{C_{min}}{C_h} (T_{h1} - T_{c1}) + \frac{1}{C_c} = e^{-NTU \left( 1 + \frac{C_{min}}{C_{max}} \right)}$$

$$or, \epsilon \epsilon = \frac{1 - e^{-NTU \left( 1 + \frac{C_{min}}{C_{max}} \right)}}{C_{min} \left( C_h + C_c \right)}$$

Now, assuming  $C_h > C_c$ , i.e. cold fluid as the 'minimum fluid', we have  $C_{min} = C_c$  and  $C_{max} = C_h$

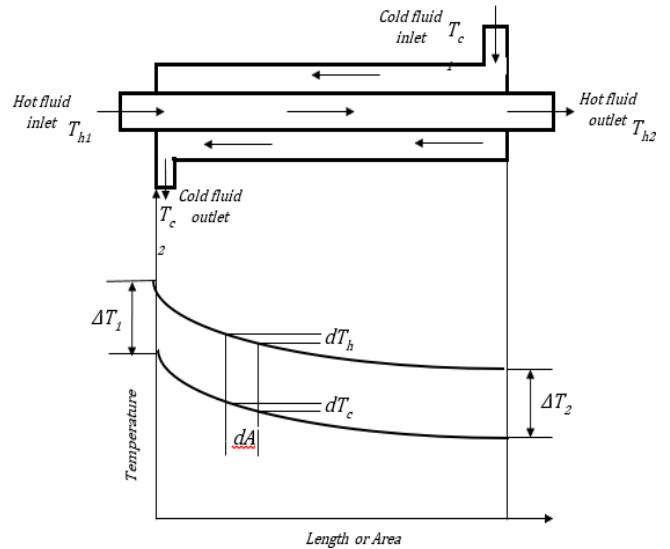
Therefore

$$\epsilon \epsilon = \frac{1 - e^{-NTU \left( 1 + \frac{C_{min}}{C_{max}} \right)}}{C_{min} \left( 1 + C_{max} \right)}$$

$$\epsilon \epsilon = \frac{1 - e^{-NTU[1+C]}}{(1+C)}$$

Where,  $C = \frac{C_{min}}{C_{max}}$

**Counter Flow Heat Exchanger:** Consider a double pipe, counter flow heat exchanger, in which a hot fluid and a cold fluid flow parallel to each other, separated by a solid wall. Hot fluid enters at a temperature of  $T_{h1}$  and leaves the heat exchanger at a temperature of  $T_{h2}$ ; cold fluid enters the heat exchanger at a temperature of  $T_{c1}$  and leaves at a temperature of  $T_{c2}$ . This situation is shown in figure.



We desire to get an expression for the rate of heat transfer in this heat exchanger in the following form:

$$Q = UA(\Delta T)_{mean} \text{----- (a)}$$

U = Overall Heat transfer coefficient

A = Area of heat transfer

$(\Delta T)_{mean}$  = Mean temperature difference between fluids.

Now, we make the following assumptions:

- i. U is considered constant throughout the length of the heat exchanger
- ii. Properties of fluids are considered constant.
- iii. Heat exchange takes place only between the two fluids.
- iv. Changes in potential and kinetic energy are negligible.
- v. Temperature of both fluids remain constant over a given cross section.

Area 'A' is constant for a given heat exchanger. However, we see from figure that the temperature of the two fluids vary along the length (or area) of the heat exchanger, i.e. the temperature difference between the hot and cold fluids is not a constant along the length of the heat exchanger, but varies along the length. Our aim is to find out the appropriate 'mean temperature difference ( $\Delta T_m$ )' between the hot and cold fluids, so that Eq. (a) can be applied. We proceed as follows:

Consider an elemental area  $dA$ , applying 1<sup>st</sup> law of thermodynamics,

Heat given up by hot fluid = Heat received by the cold fluid

$$-m_h C_{ph} dT_h = m_c C_{pc} dT_c \text{----- (b)}$$

$$dT_h = \frac{-dQ}{m_h C_{ph}} \text{ and } dT_c = \frac{dQ}{m_c C_{pc}}$$

where,  $m_h$  and  $m_c$  are the mass flow rates, and  $C_{ph}$  and  $C_{pc}$  are the specific heats of hot and cold fluids, respectively. Therefore,

$$dT_h - dT_c = d(T_h - T_c) = -dQ \left[ \frac{1}{m_h C_{ph}} - \frac{1}{m_c C_{pc}} \right]$$

Substituting for  $dQ$  from Eq. (b), we get:

$$\begin{aligned} d(T_h - T_c) &= -U(T_h - T_c)dA \left[ \frac{1}{m_h C_{ph}} - \frac{1}{m_c C_{pc}} \right] \\ \frac{d(T_h - T_c)}{(T_h - T_c)} &= -U \left[ \frac{1}{m_h C_{ph}} - \frac{1}{m_c C_{pc}} \right] dA \end{aligned}$$

Integrating between inlet and exit of the heat exchanger:

$$\ln \left[ \frac{T_{h2} - T_{c2}}{T_{h1} - T_{c1}} \right] = -UA \left[ \frac{1}{m_h C_{ph}} - \frac{1}{m_c C_{pc}} \right]$$

Now, out of the two fluids, one is the 'minimum' fluid and the other is the 'maximum' fluid.

Whichever may be the minimum fluid, we can write the above equation as:

$$\ln \frac{T_{h2} - T_{c2}}{T_{h1} - T_{c1}} = -UA \left( \frac{1}{C_{min}} - \frac{1}{C_{max}} \right)$$

$$= -\frac{UA}{C_{min}} \left( 1 - \frac{C_{min}}{C_{max}} \right)$$

$$\frac{T_{h2} - T_{c2}}{T_{h1} - T_{c1}} = e^{-\frac{UA}{C_{min}} \left( 1 - \frac{C_{min}}{C_{max}} \right)}$$

Now we know that,

$$\epsilon \epsilon = \frac{dd_{act}}{dd_{max}} = \frac{C_h(T_{h1} - T_{h2})}{C_{min}(T_{h1} - T_{c1})} = \frac{C_h(T_{c2} - T_{c1})}{C_{min}(T_{h1} - T_{c1})}$$

$$\Rightarrow T_{h2} = T_{h1} - \epsilon \epsilon \frac{C_h}{C_h} (T_{h1} - T_{c1})$$

$$T_{c2} = T_{c1} + \epsilon \epsilon \frac{C_{min}}{C_c} (T_{h1} - T_{c1})$$

Now substituting the values of  $T_{h2}$  and  $T_{c2}$  in the equation

$$\frac{T_{h2} - T_{c2}}{T_{h1} - T_{c1}} = e^{-\frac{UA}{C_{min}} \left( 1 - \frac{C_{min}}{C_{max}} \right)}$$

$$\frac{T_{h1} - \epsilon \epsilon \frac{C_{min}}{C_h} (T_{h1} - T_{c1}) - T_{c1}}{T_{h1} - \epsilon \epsilon \frac{C_{min}}{C_c} (T_{h1} - T_{c1}) - T_{c1}} = e^{-\frac{UA}{C_{min}} \left( 1 - \frac{C_{min}}{C_{max}} \right)}$$

Now let,  $NTU = \frac{UA}{C_{min}}$  and  $C_h > C_c$ , i.e. cold fluid as the 'minimum fluid', we have  $C_{min} = C_c$  and  $C_{max} = C_h$

$$\frac{(T_{h1} - T_{c1})(1 - \epsilon \epsilon)}{(T_{h1} - T_{c1})(1 - C \epsilon \epsilon)} = e^{-NTU[1-C]}$$

$$\frac{(1 - \epsilon \epsilon)}{(1 - C \epsilon \epsilon)} = e^{-NTU[1+C]}$$

$$\text{or, } \epsilon \epsilon = \frac{1 - e^{-NTU[1-C]}}{1 - C e^{-NTU[1-C]}}$$

### Special cases:

(i) **For a condenser or boiler:** i.e. one of the fluids undergoes a phase change. Therefore,  $C_{max} \rightarrow \infty$  i.e. Capacity ratio,  $C = 0$ . Then effectiveness relation (for all heat exchangers) reduces to:

$$\epsilon \epsilon = 1 - e^{-NTU}$$

(ii) **When  $C = 1$ , i.e.  $C_{min} = C_{max}$ :** This is the case of a typical, gas turbine regenerator. In this case, relation for  $\epsilon$  reduces to the indeterminate form,  $0/0$ . Then, apply the L' Hospital's rule to evaluate  $\epsilon$ . i.e. differentiate the numerator and denominator w.r.t.  $C$  and taking the limit  $C \rightarrow 1$ , we get:

$$\epsilon \epsilon = \frac{NTU}{1 + NTU}$$

**Compact Heat Exchangers:** Heat exchangers with an area density greater than about  $700 \text{ m}^2/\text{m}^3$  are classified as 'compact heat exchangers'.

Generally, they are used for gases.

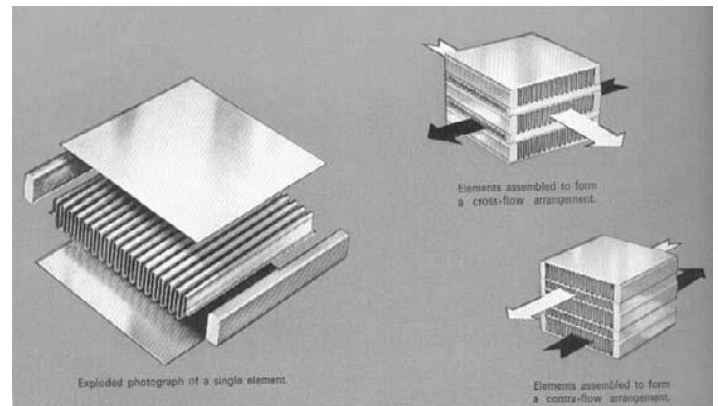
Compact heat exchangers are, typically, of three types:

- i. array of finned circular tubes
- ii. array of plate-fin matrix, and
- iii. array of finned flat-tube matrix.

Heat transfer and pressure drops for these compact heat exchangers are determined experimentally and are supplied by manufacturers as their proprietary data.

As an example, a plate-fin type of heat exchanger matrix, manufactured by Marston-Excelsior Ltd., is shown in figure. As shown in the figure, a single element consists of two plates in between which is sandwiched a corrugated sheet. The two edges are sealed. Dip brazing technique is used to build a complete heat exchanger block from individual elements. Multi-flow configurations are possible, and the generally

used corrugations types are: plain (P), plain-perforated (R), serrated (S) and herringbone (H).



**Module 5: HEAT TRANSFER WITH PHASE CHANGE**

There are many important practical cases which involve heat transfer with a change of phase of the fluid, e.g. boiling where the liquid changes to vapour and condensation where the vapour condenses into a liquid. Boiling and condensation are classified under convection since there is motion of the fluid during heat transfer in these processes.

Some of the applications of boiling and condensation are:

- (i) Evaporators and condensers of a vapour compression refrigerating system
- (ii) Boilers and condensers of a steam power plant
- (iii) Reboilers and condensers of distillation columns of cryogenic and petrochemical plants
- (iv) Cooling of nuclear reactors and rocket motors
- (v) Process heating and cooling, etc.

**BOILING HEAT TRANSFER**

**Boiling and Evaporation:** 'Boiling' occurs at the solid–liquid interface when the solid surface is at a temperature  $T_s$ , sufficiently above the saturation temperature  $T_{sat}$  of the liquid at that pressure. In contrast, 'evaporation' occurs at the liquid–vapour interface when the vapour pressure above the liquid is less than the saturation pressure of the liquid at the given temperature.

**Boiling Modes:**

Boiling is generally classified as 'pool boiling' and 'flow boiling'.

In pool boiling, there is no bulk fluid flow, and any motion of the fluid is due to natural convection and the movement of bubbles under buoyancy effects. Heating of a liquid by immersing a heating element in it is an example of pool boiling. When boiling occurs while fluid is in motion under the influence of a pump, it is called flow boiling.

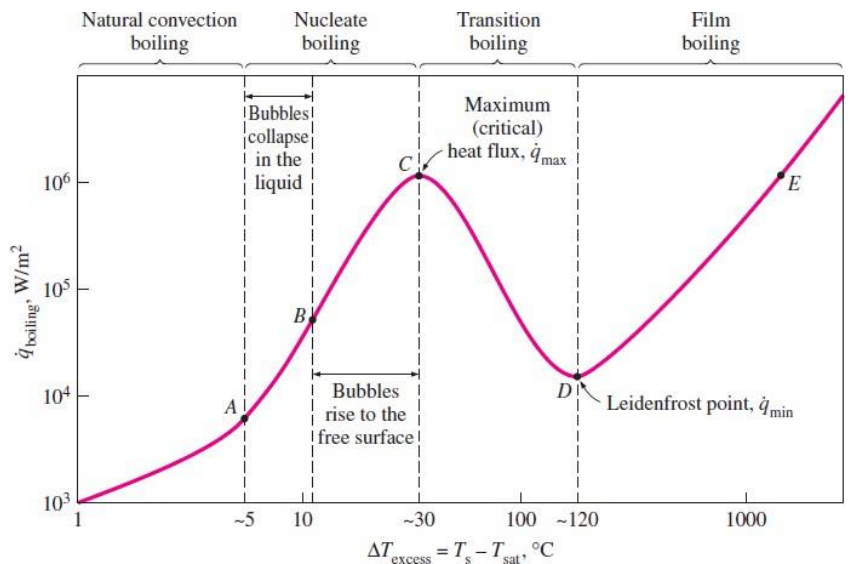
These two modes of boiling are further classified as 'sub-cooled boiling' and 'saturated boiling'. In sub-cooled boiling, main body of the liquid is at a temperature below the saturation temperature  $T_{sat}$ , while in saturated boiling, main body of the liquid is at a temperature equal to  $T_{sat}$ .

**Boiling Regimes and Boiling Curve:**

Nukiyama performed his pioneering experiments on boiling heat transfer in 1934. He used nichrome and platinum wires which were electrically heated while immersed in liquids. In general, four different boiling regimes are observed depending upon the excess temperature ( $\Delta T_e$ ) imposed, namely

- (i) Natural convection boiling ( $\Delta T_e$  upto about 5 deg.C)
- (ii) Nucleate boiling ( $\Delta T_e$  from 5 deg to about 30 deg.C)
- (iii) Transition boiling ( $\Delta T_e$  from 30 deg to about 120 deg.C), and
- (iv) Film boiling ( $\Delta T_e$  beyond 120 deg.C)

The figure shows a typical boiling curve for water at one atmosphere pressure. General shape of the boiling curve is same for other fluids as well. In the figure, boiling heat flux is plotted against the excess temperature. Also, shape of the boiling curve is independent of the geometry of the heating surface, but depends on the fluid pressure and the specific fluid–heating surface combination.



i) **Natural convection boiling:** This range is up to the point 'A' in the figure. No bubbles are formed up to a small excess temperature of about 5 deg. and the liquid is superheated, rises to the free surface and evaporates from the surface. In this range, the free convection correlations can be applied to make heat transfer calculations.

ii) **Nucleate boiling:** Region between 'A' and 'C' is the nucleate boiling region. Starting from point 'A', as  $\Delta T_e$  increases, bubbles start forming at nucleation sites at an increasing rate. The figure shows the nucleate boiling of methanol on a horizontal surface. Nucleate boiling region may be classified into two sub-regions:

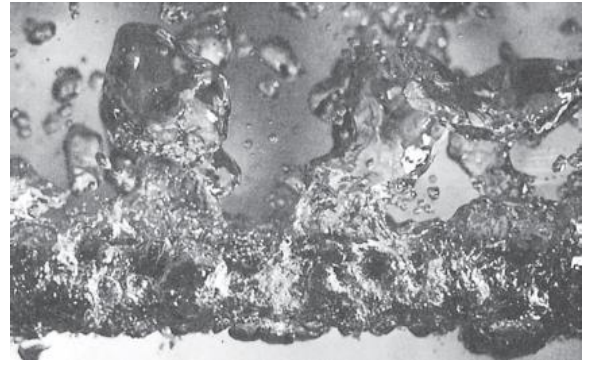


(a) region A–B, where the isolated bubbles formed rise up, but do not reach the free surface and collapse in the body of the liquid; space vacated by the bubbles formed at the surface as they move up, is filled by fresh liquid, and the process is repeated. Movement of the bubbles through the body of the liquid causes agitation which is responsible for increasing heat transfer in nucleate boiling.

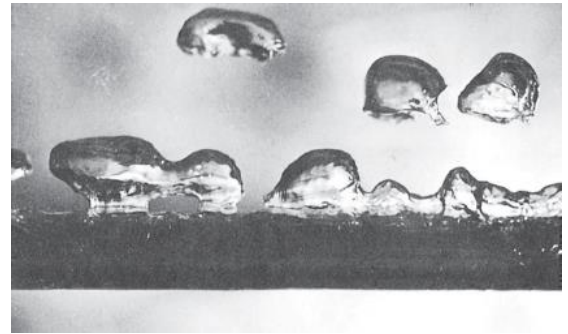
(b) region B–C, where the bubbles form at a faster rate at a largely increased number of nucleation sites and rise up in the liquid in almost continuous columns of vapour. These bubbles gush up in the liquid and reach the free surface and then collapse. Heat flux in this region is very large due to this reason. Note that after point B there is an inflection in the boiling curve; this is because of the fact that as excess temperature is increased, the heating surface gets almost covered with bubbles and the heat flux increases at a lower rate as  $\Delta T_e$  increases, and reaches a maximum at point C. Heat flux at point C is called 'critical' or 'maximum' or 'burnout' heat flux,  $q_{\text{max}}$ . For water,  $q_{\text{max}} > 1 \text{ MW/m}^2$ . It should be clear that from heat transfer point of view, nucleate boiling regime is the most desirable range to operate, since very high heat transfer rates are obtained with relatively small  $\Delta T_e$  (under  $30^\circ\text{C}$ )



**iii) Transition boiling:** Region between 'C' and 'D' is the transition boiling region. In this range, as the excess temperature increases, the heat flux decreases; this is due to the fact that now a major portion of the heater surface is covered by the vapour film which has a smaller thermal conductivity as compared to that of the liquid, and, therefore, acts as an insulation. Between points C and D, nucleate and film boiling occur partially or alternately and is therefore called 'unstable film boiling regime'. At point D, excess temperature is of the order of  $120^{\circ}\text{C}$ . The figure shows the transition boiling of methanol on a horizontal surface.



**iv) Film boiling:** This region is beyond the point D. As excess temperature is further increased, now a stable, vapour blanket completely covers the heater surface. So, at point D, the heat flux reaches a minimum and this point is known as 'Leidenfrost point', (in honour of Leidenfrost, who explained in 1756 that the water drops dropped on a very hot surface 'dance' on a vapour film and boil away).



Now, as the excess temperature is increased further, heat transfer by radiation effect also comes into picture in addition to conduction through the vapour film, and the heat flux increases as shown. The figure shows the film boiling of methanol on a horizontal surface.

### Critical Heat Flux in Nucleate Pool Boiling:

The **burnout phenomenon** in boiling can be explained as follows: In order to move beyond point C where  $q_{max}$  occurs, we must increase the heater surface temperature  $T_s$ . To increase  $T_s$ , however, we must increase the heat flux. But the fluid cannot receive this increased energy at an excess temperature just beyond point C. Therefore, the heater surface ends up absorbing the increased energy, causing the heater surface temperature  $T_s$  to rise. But the fluid can receive even less energy at this increased excess temperature, causing the heater surface temperature  $T_s$  to rise even further. This continues until the surface temperature reaches a point at which it no longer rises and the heat supplied can be transferred to the fluid steadily. This is point E on the boiling curve, which corresponds to very high surface temperatures. Therefore, any attempt to increase the heat flux beyond  $q_{max}$  will cause the operation point on the boiling curve to jump suddenly from point C to point E. However, surface temperature that corresponds to point E is beyond the melting point of most heater materials, and *burnout* occurs. Therefore, point C on the boiling curve is also called the **burnout point**, and the heat flux at this point the **burnout heat flux**.

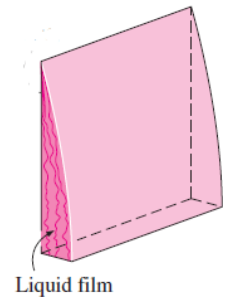
Most boiling heat transfer equipment in practice operate slightly below  $q_{max}$  to avoid any disastrous burnout. However, in cryogenic applications involving fluids with very low boiling points such as oxygen and nitrogen, point E usually falls below the melting point of the heater materials, and steady film boiling can be used in those cases without any danger of burnout.

## CONDENSATION

Condensation is defined as the removal of heat from a system in such a manner that vapour is converted into liquid. When the temperature of a vapour is decreased below its saturation temperature, the vapour condenses. After coming into contact with a cool surface, the vapour's latent heat is released, and heat is transferred to the surface, resulting in the formation of condensate. The condensate will accumulate on the horizontal surface, till the whole surface is covered by the liquid. On a vertical or inclined surface, however, the condensate will flow downwards along the surface under the influence of gravity.

The vapours may condense on the surface in one of the two modes: 'film-wise condensation' or 'drop-wise condensation'.

In **film wise** condensation, the dominant mode, a thin continuous liquid film covers the entire condensing surface and the condensate flows off the surface under the action of gravity. This occurs when the surface is clean and uncontaminated, and the vapour is relatively free of impurities. In the film-condensation process, the surface is covered by the film, which grows in thickness as it moves down the surface. The presence of a liquid film over the surface constitutes thermal resistance to heat transfer.



The second type of condensation, known as **dropwise condensation** is observed to occur when the surface is coated with a substance that inhibits wetting or is contaminated with oil or other fatty acids or is highly polished. In this case, individual drops are formed on the condensing surface. These drops grow in size and coalesce (combine with one another), as they roll down in some random fashion, to form larger ones, thus leaving the surface exposed for the formation of a new drop. In dropwise condensation, a significantly large part of the surface is directly exposed to the vapour. Of the two, higher condensation and heat-transfer rates are experienced in dropwise condensation. For this reason, many surface coatings and vapour additives like oleic acid have been used to promote and maintain dropwise condensation. There is no film barrier to heat transfer in dropwise condensation, and a portion of the cool surface is always in contact with the vapour without the insulating influence of the liquid layer. This accounts for higher heat-transfer coefficients (up to 290 kW/m<sup>2</sup>K) associated with dropwise condensation which is certainly preferable for industrial applications. However, commercially, dropwise condensation is difficult to attain and maintain for long periods of time (even the promoters employed to prolong this mode lose their effectiveness in course of time). Hence, all condensing equipments are usually designed by assuming that film condensation will exist.

